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ON THE SYSTEMATIC MOTIONS OF THE K-STARS.

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DOCTOR IN DE WIS- EN STERRENKUNDE AAN DE
RIJKS-UNIVERSITEIT TE GRONINGEN, OP GEZAG VAN DEN
RECTOR-MAGNIFICUS Dr. J. VAN WAGENINGEN, HOOGLEERAAR
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AAN MIJNE OUDERS.

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De voltooiing van dit proefschrift biedt mij de welkome gelegenheid U, mijne Heeren Hoogleeraren in de Faculteit der Wis- en Natuurkunde, die tot mijne academische vorming hebben bijgedragen, mijn hartelijken dank te betuigen.

Ook U, Hooggeleerde ORNSTEIN, *wil ik daarbij niet vergeten; terwijl de herinnering aan de reeds ontslapen leden der Faculteit* F. DE BOER, P. H. SCHOUTE *en* F. J. P. VAN CALKER *bij mij steeds levendig zal blijven.*

Bovenal zij het mij vergund een afzonderlijk woord van dank te richten tot U, Hooggeleerde KAPTEYN, *Hooggeachte Promotor. Uwe lessen, die mij destijds reeds naar Groningen deden komen, versterkten in mij mijn voorliefde voor de sterrenkunde. En later, temidden Uwer vele bezigheden, waren het Uwe steeds onverflauwde belangstelling en aanmoediging, die mij mijn proefschrift deden voltooien.*

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ERRATUM:

Page 4 note, for 37 read: 67.

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CHAPTER I.

AIM OF THE INVESTIGATION.

By the researches of this time about the structure of the universe it has been found that the division of the stars into spectral classes is not only of interest for their physical condition, but that there is also a conspicuous correlation between spectral type and other properties of the stars.

In the first place we may expect a relation between spectral type and luminosity. By examining lists of stars in Publications of the Astronomical Laboratory at Groningen 24 from Kapteyn and Weersma Eddington 1) found that "they show a remarkable tendency towards equality of brightness among stars "of the same type and that there is a striking progressive diminution of brightness "with advance in the stage of evolution." Though at the present moment this conclusion might seem to be somewhat too general, still it is undoubtedly true that there is some correlation between spectral type and brightness, at least for the averages of larger groups of stars. As regards proper motion, both Monck 2) and Kapteyn 3) found that the stars of type II have a larger velocity than those of type I. Afterwards Kapteyn 4) examined all available radial velocities and indicated that there appears to be "a regular increase in the peculiar radial "velocity, when we pass from the Orion (*B*) to the Sirian (*A*) stars, from these "to the second type stars (*F* to *K*), finally to those of the third type (*M*)." Independent of Kapteyn, Campbell 5) came to the same conclusion, viz.: "the "progression of average velocity with advancing spectral type is clear and "unmistakable." 6)

Neither is the degree of condensation to the galactic plane the same

---

1) Eddington. Stellar movements p. 48, 1914.

2) Astronomy and astrophysics **11** and **12**, 1892.

3) Astron. nachr. **3487** 1898.

4) Astrophys. Journ. **31**, 259, 1910.

5) Lick Bulletin **196** 1911.

6) Campbell Stellar motions, p. 206, 1913.

for the different types. It is a fact that all stars show condensation with decreasing galactic latitude, but this condensation is for the *B* and *A* stars much more striking than for the stars of the older types. This does not apply to *B* and *A* stars with large proper motion, as these show hardly any condensation to the Milky way.

Lastly, there apparently is a relation between spectral type and star-streaming. About this Eddington [1] writes: "It had been found by many independent researches that the star-streaming tendency is scarcely shown in the type "*B* stars, that it is most strongly shown in type *A* and that it becomes less "marked in succeeding types, though still quite prominent in type *K*."

We see that all these results indicate that it is urgently necessary to examine the arrangement of the stars for each spectral class separately and not only this, but also, if it is possible, for each subdivision, in order to get a better insight into the now more or less hypothetical relation between the spectral classes and the properties mentioned above.

Owing to want of material, it is impossible as yet to examine the subdivisions of the spectral classes, so that we must be content with a discussion of the classes themselves

In the following pages we have tried to give a contribution to this general investigation. For this purpose we have chosen to examine for stars of type *K*:

1) their vertices.
2) their velocities in the two star-streams.
3) their numeric distribution over the two streams and
4) in connection with this:
   the motion of the sun with relation to the centre of gravity of the stars investigated.
5) the velocity of the sun by means of the radial velocities of *K*-stars.

That we have taken stars of type *K* is owing to the circumstance that the available number of stars of this type is larger than the number of stars of other types in the same catalogue, except those of type *A*, but for the latter something is known already about the distribution of the stars over the two streams, viz. that the proportion is 1 : 3. It is commonly assumed that among the Helium stars there are no members of the second stream at all. Though this seems exaggerated [2], still the proportion can hardly be below 1 to 15 or 1 to 30.

As it will be of some importance in this connection to examine the manner of distribution for stars of type *K*, we have chosen this class for our purpose.

---

[1] Eddington. Stellar movements p. 162, 1914.

[2] Kapteyn. Handel. van het 13e Ned. Nat. en Geneesk. congr. p. 28, 1911.

CHAPTER II.

CHOICE OF METHOD.

It is not easy to make a choice between the different methods which can be used for the investigation mentioned above. No fewer than 4 well-known methods viz. those of Kapteyn, Eddington, Dyson and Schwarzschild are at our disposal.

The oldest theory is that of Kapteyn [1]. As early as 1904 he published the results, which he had obtained by means of this method. The theory itself was not published until 1912. This had its reason in the fact, that Kapteyn thought his theory a more or less "cumbrous one, which leads to very laborious "calculations as soon as we wish to make a detailed comparison with the obser-"vations."

Another objection to its application is the assumption of equality in number of stars of the two star-streams, while we have reason to assume that the number of stars are as a rule unequal. As, however, we know that at least for the B and A stars equality of number does not exist, it seems risky to make such an assumption for any one single class.

The method of Dyson [2] published in 1907 is the most simple of all, because it is independent of any hypothesis of the distribution of velocities. Dyson's method is only based on the assumption that the observed curves of the proper motions are compounded of two single drift curves. Graphically, without any mathematical theory, he makes two symmetrical curves which together represent the observed curve. Consequently the mathematical constants of the other methods are not present, so that it does not give an easy general view of all the circumstances, but only the two directions of the streams.

Schwarzschild [3] was not content that the system of stars had been divided into two systems, which, like gases of stars, penetrate each other entirely.

---

[1] Monthly notices 72, 1912.

[2] Proc. of the Royal Soc. of Edinburgh 28, 1907.

[3] Nachrichten von der Kgl. Gesellsch. der Wiss. zu Göttingen, p. 614, 1907.

Therefore he has given a theory based on the unity of the structure of the Universe, which he published in 1907.

This theory replaces the distribution of velocities according to the Maxwellian law by an ellipsoidal distribution.

The same objection which was made to Kapteyn's theory must also be mentioned here viz. the number of stars moving in the two favoured directions is equal.

Lastly the method of Eddington ¹), which he published in 1906. It is most fit for our purpose.

The defects of the other methods are not here. It does not start from an hypothesis of the numbers of stars belonging to the two streams, and the mathematical theory and calculations are not too difficult in the application. It is graphical for the greater part and yet it gives the necessary constants.

It is true that there are also objections to this method. In the first place, it does not use the amount of the velocities, but only the number of proper motions, but both Dyson and Schwarzschild also neglect it. Only Kapteyn is accurate in this respect. In the second place, it is based on the Maxwellian law, but as Kapteyn's as well as Schwarzschild's method have this drawback, this only makes the choice between Eddington's and Dyson's method more difficult.

Taking everything into consideration, Eddington's method seems preferable to the other three for the present investigation.

---

¹) Monthly notices **37**, Nov. 1906.

CHAPTER III.

DISCUSSION OF EDDINGTON's METHOD.

a. Introduction.

We consider a small region of the sky, so that it may be treated as a plane tangential to the sky in the centre of the region

Of a number of stars in that region we know the direction of the proper motion, and as a matter of course the *R. A.* and the declination. By the direction of the proper motion we understand the direction of the projection of the proper motion on the sky i. e. on the tangential plane.

All these directions we indicate by arrows and these arrows we move parallel to themselves to the centre of the region.

If the stars do not prefer any direction of motion, the arrows will be regularly divided over the 360° encircling the centre.

If there is one preferred direction, the number of arrows will be divided bilaterally symmetrical relative to this direction.

If there are two preferred directions, there will be a number of arrows arranged relative to the one and another number arranged relative to the other direction.

To express it more precisely, we will divide the 360° into the angles 0° to 10°, 10° to 20° etc. In the directions 5°, 15°, 25° etc. we draw radii vectores proportional to the *numbers* of arrows respectively between 0° and 10°, 10° and 20° etc. Combining the tops of these radii vectores by a smoothed line, we shall get

1) a circle, when there is no preferred direction.

2) a curve bilateral symmetrical, when there is one preferred direction.

3) a figure with two maxima, when there are two preferred directions.

We call these curves *proper motion curves.*

We shall see that we obtain proper motion curves which approximately represent the last case. But as the number of stars is comparatively small, the figures are very irregular, so that it is difficult to say something about the preferred directions with any precision. Moreover, even if the curves were regular with two well-defined maxima, they would not indicate the preferred directions, as the number of proper motions in each direction of one drift influence the direction of the maximum of the other drift

For this reason Eddington has devised a mathematical theory, which enables us to form theoretical curves, which we can compare with the irregular curves directly obtained from the observation.

The form of these theoretical curves can be altered by giving other values to some constants.

Now our intention is to choose these constants in such a way that we get theoretical curves which agree as nearly as possible with the curves from the observation.

From these constants, of which we shall have two groups of three for each figure, we can easily derive some data about:

1) the directions of the two star-streams.
2) the speed.
3) the proportion of the stars belonging to the two star-streams.

b. Hypothesis of Eddington

In order to derive his theory, Eddington introduces a supposition in regard to the motion of the stars.

The motion of a star in space can be divided into two components.

One component is the motion of the centre of gravity of the system of stars to which the star belongs. The other component is the motion of a star relative to axes of coordinates, which move with the same velocity and in the same direction as the centre of gravity of the system.

We call the second component of the proper motion of a star its "peculiar" motion.

Because Eddington wishes to give a mathematical form to the distribution of the peculiar motions of all the stars of the system, he compares the stars of the system with the molecules of a gas. He supposes that the peculiar velocities of the stars obey the same law as these molecules. In accordance with the dynamical theory of gases he gives for the distribution of the peculiar velocities the following hypothesis:

The distribution of the peculiar velocities of the stars of a system will be in accordance with the law of Maxwell.

i. e.:

a) The peculiar motions of the stars have no preference for any particular direction. Their directions are quite at random.

b) There will be a mean peculiar speed The deviations of the motions from the amount of this speed will be in accordance with this Maxwellian or error-law.

In imitation of Eddington we call such a system of stars a "drift", and its motion "drift motion".

c. Objections to the hypothesis.

The application of this law to the peculiar motion of the stars is not without objections.

It is a fact that the stars are not to be compared with the molecules of a gas. The kinematical theory of gases is based on an infinite number of mutual collisions of the molecules. It can hardly be imagined that such a thing or even anything on the same scale should happen with the stars. Besides, investigations have proved that the law does not hold good for the stars. Schwarzschild [1]) has derived a distribution of velocities using values relative to the sun. This distribution of velocities has been compared by him with the distribution of velocities according to Maxwell's law. To be able to make this comparison, he changed Maxwell's distribution of velocities, which only refers to the peculiar proper motions, by adding to these peculiar proper motions the parallactic motion. A comparison of the two distributions showed, that there are fewer average proper motions, but more large proper motions than Maxwell's distribution gives.

Kapteyn and Adams [2]) come to a similar conclusion, when they derive the relations between the proper motions and the radial velocities of the stars of the spectral classes *F*, *G*, *K* and *M*. They find among others that "for all of "the spectral classes the average radial velocities show a regular increase with "the proper motion" and "such a change of radial velocity is a necessary consequence of a velocity distribution (for the peculiar motions) different from that given by Maxwell's law."

d. The application of the Maxwellian distribution of velocities to the stars.

In the first place we make the following suppositions:

We consider a small region of the sky, and in the centre of that region a tangential plane to the sky.

---

[1]) Astron. Nachr. **4557**, 1912.

[2]) Proc. Nat. Acad. of Science. Boston **1**, 14, 1915.

Of the velocities of the stars in that region we only take those components which are perpendicular to the line of sight and suppose that these components are lying in the tangential plane.

Therefore on one hand the region must be small, that it does not differ too much from the tangential plane; on the other hand it must be so large that the number of stars in the region is not too small for the application of the theory.

We take the *linear* velocities of the stars.

We suppose that all the stars of that region form one drift with a linear velocity V relative to the observer. This velocity is directed to a fixed point in space. Let λ be the angle between the line of sight to the centre of the region and the direction of the velocity V i. e. the distance from the centre of the region to the "antivertex" [1]) of the drift of stars. The component of V perpendicular to the line of sight is $V \sin \lambda$.

We make in the tangential plane rectangular axes of coordinates. Let the abscis be along the direction towards the antivertex.

Except the velocity $V \sin \lambda$, each star has in the tangential plane a velocity with components u and v, respectively along ordinate and abscis.

We apply the law of Maxwell to these components u and v and suppose that there is a total number of stars in the considered region $= N$.

We then find:

The number of stars having component peculiar velocities between (u, v) and $(u + du, v + dv)$

$$N \left(\frac{h}{\sqrt{\pi}} \right)^2 e^{-h^2 (u^2 + v^2)} du\, dv.$$

h, the modulus of precision, is connected with the mean peculiar speed Ω of the stars of the drift. This mean peculiar speed is

$$\Omega = \frac{1}{h} \sqrt{\frac{4}{\pi}}.$$

This formula gives the number of stars having velocities between (u, v) and $(u + du, v + dv)$.

Our aim is to find a formula which gives us the *number* of stars which have their velocities in a definite direction, when we compose the velocity $V \sin \lambda$ with the components u and v of the peculiar velocities.

So we wish to derive the number of stars having the projections of their proper motions on the tangential plane in the direction between θ and $d\theta$, if θ indicate the inclination to the X-axis.

---

[1]) Corresponding to the names apex and antapex of the Sun's motion, where apex denotes the point of divergence and antapex the point of convergence of the stars, we shall use with reference to the directions of the starstreams the term *vertex* for the point where the stars *diverge* and *antivertex* for the point where they *converge*.

In order to get this, we shall use polar coordinates.

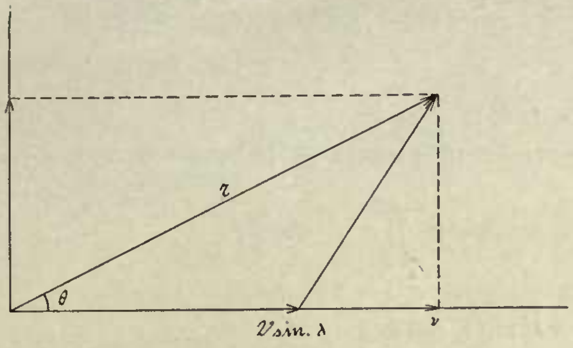

Fig. I.

$$u^2+v^2=r^2+(V\sin\lambda)^2-2\,V\sin\lambda\, r\cos\theta$$
$$du\,dv = r\,dr\,d\theta.$$

By substituting this in the formula:

$$n = N\left(\frac{h}{\sqrt{\pi}}\right)^2 e^{-h^2(u^2+v^2)}\,du\,dv$$

we get:

number of stars having velocity between r and $r+dr$ in directions between θ and $\theta+d\theta$

$$n = N\left(\frac{h}{\sqrt{\pi}}\right)^2 e^{-h^2\{r^2+(V\sin\lambda)^2-2V\sin\lambda\, r\cos\theta\}}\,r\,dr\,d\theta.$$

As we wish to eliminate the amount of the velocities, this must be integrated over r from 0 to ∞:

number of stars having an arbitrary velocity in directions between θ and $\theta+d\theta$

$$= N\left(\frac{h}{\sqrt{\pi}}\right)^2 d\theta\int_0^\infty e^{-h^2(r^2+V^2\sin^2\lambda-2V\sin\lambda\, r\cos\theta)}\,r\,dr.$$

Substituting $z = r - V\sin\lambda\cos\theta$, so $dz = dr$ $(\theta = \text{constant})$ and $z^2+V^2\sin^2\lambda\sin^2\theta = r^2-2Vr\sin\lambda\cos\theta+V^2\sin^2\lambda(\sin^2\theta+\cos^2\theta)$ we obtain:

$$N\left(\frac{h}{\sqrt{\pi}}\right)^2 d\theta\int_{-V\sin\lambda\cos\theta}^{\infty} e^{-h^2(z^2+V^2\sin^2\lambda\sin^2\theta)}\,(z+V\sin\lambda\cos\theta)\,dz$$

and therefore, writing $hz = x$ we find:

$$N\left(\frac{h}{\sqrt{\pi}}\right)^2 e^{-h^2V^2\sin^2\lambda\sin^2\theta}\,d\theta\int_{-hV\sin\lambda\cos\theta}^{\infty} e^{-x^2}\left(\frac{x}{h}+V\sin\lambda\cos\theta\right)\frac{1}{h}\,dx.$$

Now, when τ is written for $hV\sin\lambda\cos\theta$,

the form $e^{-h^2V^2\sin^2\lambda\sin^2\theta}$ becomes $e^{-h^2V^2\sin^2\lambda}\times c^{\tau^2}$

and thus, by substituting, the number of stars becomes:

$$N\left(\frac{h}{\sqrt{\pi}}\right)^2\frac{1}{h^2}e^{-h^2V^2\sin^2\lambda}\,d\theta\left\{e^{\tau^2}\int_{-\tau}^{\infty}e^{-x^2}(x+\tau)\,dx\right\}$$

or

$$N\frac{1}{\pi}e^{-h^2V^2\sin^2\lambda}\,d\theta\left\{-\frac{1}{2}e^{\tau^2}\int_{-\tau}^{\infty}e^{-x^2}d(-x^2)+\tau e^{\tau^2}\int_{-\tau}^{\infty}e^{-x^2}dx\right\}.$$

Integrating the first of these, we find:
the number of stars having proper motions in directions between θ and $\theta+d\theta$

$$=N\frac{1}{\pi}e^{-h^2V^2\sin^2\lambda}\,d\theta\left\{\frac{1}{2}+\tau e^{\tau^2}\int_{-\tau}^{\infty}e^{-x^2}dx\right\},$$

where $\tau = hV\sin\lambda\cos\theta$.

The function of τ can be divided into

$$\tau e^{\tau^2}\int_{0}^{\infty}e^{-x^2}dx+\tau e^{\tau^2}\int_{0}^{\tau}e^{-x^2}dx.$$

Now we have by a known theorem:

$$\int_{0}^{\infty}e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}$$

and as $\int_{0}^{\tau}e^{-x^2}dx$ is to be computed by means of one of the numerous tables in existence,

we find, by substituting,
the number of stars having proper motions in directions between θ and $\theta+d\theta$:

$$N\frac{1}{\pi}e^{-h^2V^2\sin^2\lambda}\,d\theta\left\{\frac{1}{2}+\tau e^{\tau^2}\frac{\sqrt{\pi}}{2}+\tau e^{\tau^2}\int_{0}^{\tau}e^{-x^2}dx\right\},$$

where $\tau = hV\sin\lambda\cos\theta$
$N =$ the total number of stars of a drift in the considered region of the sky.

With the aid of this formula we can now determine the theoretical numbers of stars having proper motions in each direction.

For the same region $hV\sin\lambda$ will be a constant, as $V\sin\lambda$ is a component of the velocity of the whole drift and h is connected with the mean peculiar speed of the stars of the drift.

Now, if we give a definite value to $hV\sin\lambda$, the only variable is θ. So we find that the number of proper motions in each direction only depends on the direction itself.

The value of $hV\sin\lambda$ is unknown however. We will successively give it the most plausible values. The amount of these values is dependent on the ratio of the speed of the drift and their mean peculiar speed. For the mean peculiar speed is:

$$\Omega = \frac{1}{h}\sqrt{\frac{4}{\pi}} \text{ or } h = \frac{1}{\Omega}\sqrt{\frac{4}{\pi}}.$$

So we find:

$$hV = \frac{V}{\Omega}\sqrt{\frac{4}{\pi}}.$$

In the application of the method it proves not to be necessary to assume a value larger than 2 for hV.

So $hV \sin \lambda$ varies between zero and 2 and $\tau = hV \sin \lambda \cos \theta$ between -2 and $+2$.

As, however, for a value of $\tau < -1.2$ the number of stars moving in the corresponding direction is practically zero, these values of τ may be neglected.

Each value of $hV \sin \lambda$ will give an other distribution of the number of stars.

In order to obtain these distributions, Eddington has given the following table in which we can find the values of

$$\log f(\tau) = \log \frac{2}{\sqrt{\pi}} \left\{ \frac{1}{2} + \tau e^{\tau^2} \frac{\sqrt{\pi}}{2} + \tau e^{\tau^2} \int_0^\tau e^{-x^2} dx \right\}$$

for different values of τ.

TABLE I.

τ	$\log f(\tau)$	τ	$\log f(\tau)$	τ	$\log f(\tau)$	τ	$\log f(\tau)$
-1.3	$\bar{2}.997$	-0.4	$\bar{1}.471$	0.5	0.188	1.3	1.152
-1.2	$\bar{1}.041$	-0.3	$\bar{1}.536$	0.6	0.288	1.4	1.300
-1.1	$\bar{1}.087$	-0.2	$\bar{1}.605$	0.7	0.395	1.5	1.455
-1.0	$\bar{1}.135$	-0.1	$\bar{1}.676$	0.8	0.506	1.6	1.618
-0.9	$\bar{1}.185$	0.0	$\bar{1}.751$	0.9	0.623	1.7	1.787
-0.8	$\bar{1}.238$	0.1	$\bar{1}.830$	1.0	0.746	1.8	1.964
-0.7	$\bar{1}.292$	0.2	$\bar{1}.913$	1.1	0.875	1.9	2.148
-0.6	$\bar{1}.349$	0.3	0.000	1.2	1.010	2.0	2.339
-0.5	$\bar{1}.409$	0.4	0.091				

To find for each value of τ the value of $\log f(\tau)$, we shall have to make a graphical diagram.

But we try to find not only $\log f(\tau)$, but the value of the whole formula of the number of stars. Therefore we make one diagram for the whole calculation.

Now, the numbers of stars having proper motions in directions between θ and $\theta + d\theta$ are

$$N \frac{1}{\pi} e^{-h^2 V^2 \sin^2 \lambda} d\theta \frac{V\pi}{2} f(\tau)$$

and consequently they are proportional to

$$e^{-h^2 V^2 \sin^2 \lambda} f(\tau)$$

We calculate the value of the log of this formula and also the value of this formula itself.

Thus we find numbers proportional to the numbers of stars in question.

To find for a certain $hV \sin \lambda$ both the logarithm of the proportional numbers and the proportional numbers themselves, we have made a graphical diagram, the abscissa giving the logarithm, the ordinate the different numbers.

We have drawn a graphical line for τ belonging to $\log \tau$ and a line for $\log f(\tau)$ belonging to τ.

As $\log f(\tau)$ is augmented by $\log e^{-h^2 V^2 \sin^2 \lambda}$ and this formula varies with the value of $hV \sin \lambda$, we have drawn the curve for $\log f(\tau)$ on that side of a glass plate which touches the paper. This plate can be moved for each value of $hV \sin \lambda$ over a distance $\log e^{-h^2 V^2 \sin^2 \lambda}$. Now the curve will indicate the value of $\log e^{-h^2 V^2 \sin^2 \lambda} + \log f(\tau)$ belonging to each τ.

To find $\log \tau = \log hV \sin \lambda + \log \cos \theta$, we have made a strip of millimeter paper, which indicates the $\log \cos \theta$ ($\theta = 0^\circ\ 5^\circ\ 10^\circ\ 15^\circ \ldots 180^\circ$).

This strip of paper can be moved along the abscissae over a distance agreeing with $\log hV \sin \lambda$. The marks of $\log \cos \theta$ will indicate on the abscissa the $\log \tau$.

In the same or in another diagram lines can be drawn for $\log hV \sin \lambda$ and $\log e^{-h^2 V^2 \sin^2 \lambda}$ belonging to $hV \sin \lambda$.

In this manner we can find the distribution of the proper motions over the different directions for each value of $hV \sin \lambda$.

It is not very easy, however, to compute the proportional numbers in this way. Therefore we have made another diagram with the help of the numbers obtained in the manner mentioned above. The abscissae of the diagram indicate the values of $hV \sin \lambda$, the ordinates the angles θ from $0^\circ - 180^\circ$.

The curves are equiproportional lines The figures belonging to each curve are the proportional numbers. By interpolation between these curves we can now compute the proportional numbers in two decimals, for each $hV \sin \lambda$ between 0.1 and 2.00 and for each distribution of the position angles.

This diagram clearly shows that for $\tau = hV \sin \lambda \cos \theta < -1.2$ the number of stars can be neglected.

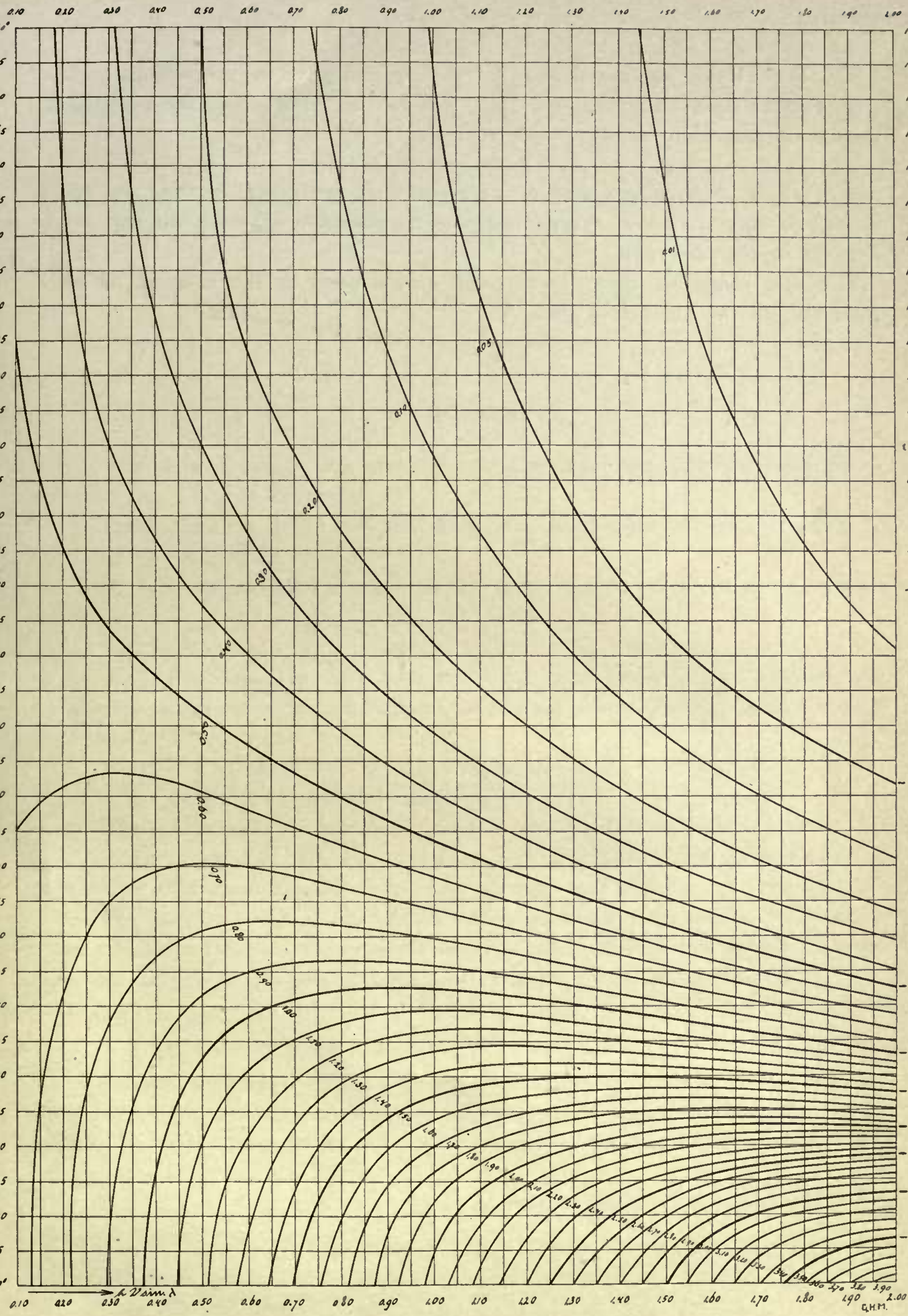

Fig. II. Diagram giving the figures proportional to the numbers of stars in each direction for $hV \sin \lambda$ between 0.10 and 2.00.

For $hV \sin \lambda = 1.2$ and $\theta = 180°$ the diagram gives the proportional number 0 0. For $hV \sin \lambda = 1.90$ and $\theta = 130°$, τ is $- 1\ 2$. Here, too, the proportional number is practically zero.

These distributions can be represented by curves by drawing the radius vector in any direction proportional to the number of stars having proper motions in that direction.

The shape of these curves will only depend on the value of $hV \sin \lambda$, their size on the number of stars N.

CHAPTER IV.

APPLICATION OF THE METHOD.

a. Materials.

The data necessary for our investigation have been derived from *Boss's Preliminary General Catalogue.*

From this catalogue all the stars have been taken which, according to *Harvard* 50, belong to type *K.*

In this manner we have got all the stars of type *K* which occur in both catalogues.

We were spared the trouble of collecting all these stars by the circumstance that Professor Kapteyn has had some lists made at the Astronomical Laboratory of Groningen in which the stars of Boss's Catalogue are found arranged according to their spectra.

From one of these lists we have taken the R. A., declination and position angle of the proper motions of the *K*-stars.

The total number of stars obtained in this manner amounts to 1420.

These stars, as will be explained in this chapter, are divided over 18 regions, so that there are on an average 79 stars per region.

We have not thought it advisable to make this number smaller by omitting those stars which have little proper motion, because indeed the number of quite unreliable angles of position is much smaller than is commonly imagined.

b. Method.

As we have seen in Chap. III *a*, we divide the sky into several regions. For each region we make a curve of the numbers of proper motions of the stars in each direction. In order to get a suitable curve, the number of stars in each region must be as large as possible. So the region may not be too small. But it is also necessary to treat the regions as planes, and this is impossible if we take our regions too large.

We should partly avoid this difficulty, if we took together the stars of two regions diametrically opposed. This treatment, however, is not desirable, as in this manner deviations, which might occur in opposite parts of the sky, would not be noticed. We shall only apply it to a few regions, which taken separately would give somewhat anomalous results.

We divide our 1420 stars over 18 regions, so that we have on an average not quite 80 stars for each region. No doubt this is a very small number, whose treatment can be defended by the results only.

The 18 regions are:

Region I	R.A. 0^h — 3^h	Decl. + 27° to — 27°	Centre R.A. 19°	Decl. + 1°
II	3^h — 6^h	⋮	69°	+ 1°
III	6^h — 9^h	⋮	111°	— 1°
IV	9^h — 12^h	⋮	156°	0°
V	12^h — 15^h	⋮	203°	0°
VI	15^h — 18^h	⋮	246°	+ 1°
VII	18^h — 21^h	⋮	287°	— 3°
VIII	21^h — 24^h	⋮	338°	— 3°
IX	0^h — 6^h	Decl. + 27° to + 63°	45°	+ 45°
X	6^h — 12^h	⋮	132°	+ 42°
XI	12^h — 18^h	⋮	223° 30′	+ 41°
XII	18^h — 24^h	⋮	313° 30′	+ 44°
XIII	0^h — 24^h	Decl. + 63° to + 90°	Centre North pole	
XIV	0^h — 6^h	Decl. — 27° to — 63°	46° 30′	— 42°
XV	6^h — 12^h	⋮	135°	— 45°
XVI	12^h — 18^h	⋮	228°	— 43°
XVII	18^h — 24^h	⋮	312°	— 41°
XVIII	0^h — 24^h	Decl. — 63 to — 90°	Centre South pole.	

The centres of these regions have been found by taking the average of the R.A. and the declinations of the stars belonging to them. This is better than taking the geometrical centres, as errors in the reduction of the angles of direction on either side of the centre will neutralize each other.

For each star the direction of the proper motion is indicated by the position-angle ¹). We remove each star with its own proper motion to the centre

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¹) We have preferred to comply with the usual method of indicating the directions of the proper motions and have not taken the angle as Eddington counts it, whose position-angles are 90° smaller. The usual position-angle is the angle between the hourcircle through the star and the direction of the proper motion counted from the hour-circle in such a way that for an angle between 0° and 90° the proper motion always gives an increasing R.A. and declination.

of the region. In doing this, the angle of position must be reduced to another hourcircle. . In the regions I to VIII the angles do not change, as the hourcircles on the tangential planes are parallel

In the regions IX to XII and XIV to XVII, which lie between + 27° and + 63°, respectively — 27° and — 63°, the angle of position is augmented or diminished by the angles between the secants of the planes of the two hourcircles and the tangential plane in the centre.

This angle has been calculated by means of the formula:

$$\operatorname{tg} \angle CPS = \operatorname{tg} (\alpha - \alpha') \sin \delta',$$

where $\angle CPS$ = the angle between the two secants,

α = the R.A. of the star,

a' and δ' = the R.A. and declination of the centre.

The regions of the North and South poles have their centres in these poles. For these regions we count the position-angle from the hourcircle belonging to a R.A. of 0° in the direction of the increasing R.A.

After all the angles of position have been determined in this manner, tables are made, giving the number of proper motions in each direction. We take together the proper motions between 0° and 10°, 10° and 20 etc. and suppose that they respectively have the directions 5°, 15° etc.

Owing to the small number of stars in each region the successive actual numbers run very irregularly. For this reason we have taken one third of the amount of three successive numbers.

As these numbers, too, are still irregular, we have applied this "smoothing" once more.

These observed numbers are to be found in the tables II to XVII, in the fifth column.

Region VI has been added to region II and region XII to region XV for a reason which will be mentioned afterwards.

The curves representing these tables have been used in the analysis.

If we compare these curves with the theoretical curves for a single drift, we shall find that they disagree too much, even if we neglect accidental irregularities.

Instead of a minimum and opposite to it a maximum there are with most of them at least two well defined maxima alternating with as many minima.

In order to represent them as accurately as possible with the help of the theoretical figures, it is necessary to take two of these theoretical curves and combine them, so that we get a single one, which represents the observed curve

As each curve depends on the two constants $hV \sin \lambda$ and N, and moreover the situation of the curves in respect to each other can be varied, there will be altogether five variable quantities. This makes it difficult to find the desired curve. They have all been found by means of "trial and error".

To obtain the compound curves the theoretical single curves are drawn on glass plates, on which a division into degrees around the origin is also made. By placing different single curves on the observed curve, two are found, which, when taken together, may be supposed to cover as nearly as possible the observed curve. The directions of the maxima of these two single curves will then form a certain angle.

To combine these two single curves and thus to find the compound curve, one of them, which we shall call curve *a*, is drawn on the glass plate on which we want to get the compound curve and on which also radii, drawn from the origin, give a division into degrees. This glass plate is laid on the other single curve, which we shall call curve *b*, the two origins covering each other, while the directions of the maxima form the right angle. The radii on the two plates will then coincide. We now move curve *a* in such a way that the outline touches the origin of curve *b*. The radii which first coincide, will now no longer correspond. But, by moving the plate parallel to itself, so that the outline of curve *a* always covers the origin of curve *b*, we now make two corresponding radii subsequently coincide.

For these two radii we can now at once mark the sum of the numbers in that direction on the glass plate.

By connecting these different points, we have obtained the compound curve.

If we lay this on the observed curve, it is easy to see which parts agree already and which disagree.

The deviations show in what manner the curve must be corrected; after repeated trials we at last get a satisfactory approximation.

All the observed curves were treated in this manner, independent of each other.

In the same way the numbers of the proper motions, to which the radii vectores of the single curves, which have been used, are proportional, were added and compared with the observed numbers.

How far theory and observation agree, can be seen in the following tables and diagrams.

TABLE II. REGION I.

ψ	Calculated numbers			Observed numbers smoothed	$o-c$
	Drift I	Drift II	Total		
5°	0.05	0.10	0.15	0.10	— 0.05
15	0.05	0.15	0.20	0.20	0.0
25	0.05	0.20	0.25	0.55	+ 0.30
35	0.05	0.30	0.35	1.10	+ 0.75
45	0.05	0.45	0.50	1.35	+ 0.85
55	0.05	0.70	0.75	1.55	+ 0.80
65	0.05	1.15	1.20	1.80	+ 0.60
75	0.05	1.80	1.85	2.35	+ 0.50
85	0.05	2.70	2.75	3.35	+ 0.60
95	0.05	3.80	3.85	4.00	+ 0.15
105	0.05	5.05	5.10	4.90	— 0.20
115	0.10	6.00	6.10	5.65	— 0.35
125	0.10	6.75	6.85	6.65	— 0.20
135	0.15	6.00	6.15	6.80	+ 0.65
145	0.25	5.05	5.30	5.45	+ 0.15
155	0.35	3.80	4.15	3.55	— 0.60
165	0.50	2.70	3.20	2.00	— 1.20
175	0.80	1.80	2.60	1.35	— 1.25
185	1.20	1.15	2.35	1.65	— 0.70
195	1.70	0.70	2.40	2.10	— 0.30
205	2.20	0.45	2.65	2.90	+ 0.25
215	2.70	0.30	3.00	3.20	+ 0.20
225	3.10	0.20	3.30	3.45	+ 0.15
235	3.10	0.15	3.25	3.20	— 0.05
245	2.70	0.10	2.80	2.20	— 0.60
255	2.20	0.10	2.30	1.45	— 0.85
265	1.70	0.10	1.80	0.90	— 0.90
275	1.20	0.05	1.25	1.10	— 0.15
285	0.80	0.05	0.85	1.00	+ 0.15
295	0.50	0.05	0.55	0.80	+ 0.25
305	0.35	0.05	0.40	0.35	— 0.05
315	0.25	0.05	0.30	0.20	— 0.10
325	0.15	0.05	0.20	0.35	+ 0.15
335	0.10	0.05	0.15	0.55	+ 0.45
345	0.10	0.10	0.20	0.55	+ 0.35
355°	0.05	0.10	0.15	0.35	+ 0.20
totals	52.30	26.90	79.20	79.0	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	125°	1.25	52
Drift II	230°	1.15	27

TABLE III. REGIONS II + VI.

ψ	Calculated numbers			Observed numbers smoothed	$o-c$
	Drift I	Drift II	Total		
5°	1.25	0.05	1.30	1.20	— 0.10
15	1.45	0.05	1.50	1.45	— 0.05
25	1.65	0.05	1.70	1.55	— 0.15
35	1.90	0.05	1.95	1.70	— 0.25
45	2.25	0.05	2.30	1.90	— 0.40
55	2.70	0.05	2.75	2.10	— 0.65
65	3.20	0.10	3.30	2.40	— 0.90
75	3.75	0.10	3.85	2.65	— 1.20
85	4.35	0.20	4.55	4.00	— 0.55
95	5.00	0.30	5.30	5.55	+ 0.25
105	5.55	0.45	6.00	7.00	+ 1.00
115	6.00	0.75	6.75	6.90	+ 0.15
125	6.20	1.25	7.45	6.55	— 0.90
135	6.20	1.95	8.15	7.00	— 1.15
145	6.00	2.85	8.85	8.45	— 0.40
155	5.55	3.65	9.20	9.90	+ 0.70
165	5.00	4.15	9.15	10.00	+ 0.85
175	4.35	4.15	8.50	8.90	+ 0.40
185	3.75	3.65	7.40	7.00	— 0.40
195	3.20	2.85	6.05	5.15	— 0.90
205	2.70	1.95	4.65	3.70	— 0.95
215	2.25	1.25	3.50	2.90	— 0.60
225	1.90	0.75	2.65	2.55	— 0.10
235	1.65	0.45	2.10	2.35	+ 0.25
245	1.45	0.30	1.75	2.00	+ 0.25
255	1.25	0.20	1.45	1.90	+ 0.45
265	1.10	0.10	1.20	1.90	+ 0.70
275	1.00	0.10	1.10	2.00	+ 0.90
285	0.95	0.05	1.00	1.75	+ 0.75
295	0.90	0.05	0.95	1.65	+ 0.70
305	0.85	0.05	0.90	1.45	+ 0.55
315	0.85	0.05	0.90	1.55	+ 0.65
325	0.90	0.05	0.95	1.10	+ 0.15
335	0.95	0.05	1.00	1.00	0.00
345	1.00	0.05	1.05	0.75	— 0.30
355°	1.10	0.05	1.15	1.15	0.00
totals	100.10	32.20	132.30	131.05	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	130°	0.55	100
Drift II	170°	1.35	32

TABLE IV. REGION III.

ψ	Calculated numbers			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.55	0.15	0.70	0.00	— 0.70
15	0.50	0.15	0.65	0.10	— 0.55
25	0.50	0.15	0.65	0.20	— 0.45
35	0.45	0.15	0.60	0.35	— 0.25
45	0.45	0.20	0.65	0.55	— 0.10
55	0.45	0.20	0.65	0.80	+ 0.15
65	0.45	0.25	0.70	1.00	+ 0.30
75	0.45	0.30	0.75	0.90	+ 0.15
85	0.45	0.35	0.80	0.80	0.00
95	0.50	0.45	0.95	0.65	— 0.30
105	0.50	0.55	1.05	0.80	— 0.25
115	0.55	0.75	1.30	1.20	— 0.10
125	0.65	1.00	1.65	2.00	+ 0.35
135	0.75	1.30	2.05	2.65	+ 0.60
145	0.85	1.75	2.60	2.90	+ 0.30
155	0.95	2.20	3.15	3.00	— 0.15
165	1.10	2.65	3.75	3.45	— 0.30
175	1.30	3.05	4.35	4.45	+ 0.10
185	1.50	3.25	4.75	4.90	+ 0.15
195	1.65	3.25	4.90	4.90	0.00
205	1.85	3.05	4.90	4.55	— 0.35
215	2.05	2.65	4.70	4.55	— 0.15
225	2.15	2.20	4.35	4.65	+ 0.30
235	2.30	1.75	4.05	4.10	+ 0.05
245	2.30	1.30	3.60	3.80	+ 0.20
255	2.15	1.00	3.15	3.20	+ 0.05
265	2.05	0.75	2.80	3.20	+ 0.40
275	1.85	0.55	2.40	2.80	+ 0.40
285	1.65	0.45	2.10	2.35	+ 0.25
295	1.50	0.35	1.85	1.80	— 0.05
305	1.30	0.30	1.60	1.45	— 0.15
315	1.10	0.25	1.35	1.35	0.00
325	0.95	0.20	1.15	1.65	+ 0.50
335	0.85	0.20	1.05	1.55	+ 0.50
345	0.75	0.15	0.90	0.90	0.00
355°	0.65	0.15	0.80	0.35	— 0.45
totals	40.00	37.40	77.40	77.85	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	240°	0.50	40
Drift II	190°	0.85	37

TABLE V. REGION IV.

ψ	Calculated numbers			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.15	0.15	0.30	0.45	+ 0.15
15	0.10	0.15	0.25	0.20	— 0.05
25	0.10	0.15	0.25	0.10	— 0.15
35	0.05	0.20	0.25	0.00	— 0.25
45	0.05	0.25	0.30	0.00	— 0.30
55	0.05	0.30	0.35	0.00	— 0.35
65	0.05	0.40	0.45	0.00	— 0.45
75	0.05	0.45	0.50	0.10	— 0.40
85	0.05	0.60	0.65	0.35	— 0.30
95	0.05	0.80	0.85	0.65	— 0.20
105	0.05	1.05	1.10	0.90	— 0.20
115	0.05	1.40	1.45	1.45	0.00
125	0.05	1.80	1.85	1.90	+ 0.05
135	0.10	2.15	2.25	2.65	+ 0.40
145	0.10	2.45	2.55	2.90	+ 0.35
155	0.15	2.65	2.80	3.00	+ 0.20
165	0.20	2.65	2.85	2.45	— 0.40
175	0.25	2.45	2.70	2.10	— 0.60
185	0.40	2.15	2.55	2.20	— 0.35
195	0.65	1.80	2.45	2.65	+ 0.20
205	1.00	1.40	2.40	3.00	+ 0.60
215	1.60	1.05	2.65	3.00	+ 0.35
225	2.30	0.80	3.10	3.45	+ 0.35
235	3.25	0.60	3.85	3.80	— 0.05
245	4.10	0.45	4.55	4.45	— 0.10
255	4.60	0.40	5.00	4.45	— 0.55
265	4.60	0.30	4.90	4.90	0.00
275	4.10	0.25	4.35	4.55	+ 0.20
285	3.25	0.20	3.45	4.20	+ 0.75
295	2.30	0.15	2.45	2.80	+ 0.35
305	1.60	0.15	1.75	1.65	— 0.10
315	1.00	0.15	1.15	0.80	— 0.35
325	0.65	0.10	0.75	0.80	+ 0.05
335	0.40	0.10	0.50	0.80	+ 0.30
345	0.25	0.10	0.35	0.80	+ 0.45
355°	0.20	0.10	0.30	0.55	+ 0.25
totals	37.90	30.30	68.20	68.05	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	260°	1.25	38
Drift II	160°	0.85	30

TABLE VI. REGION V.

ψ	Calculated numbers			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.15	0.10	0.25	1.00	+ 0.75
15	0.10	0.10	0.20	1.10	+ 0.90
25	0.10	0.10	0.20	0.90	+ 0.70
35	0.10	0.15	0.25	0.45	+ 0.20
45	0.10	0.20	0.30	0.10	— 0.20
55	0.05	0.25	0.30	0.00	— 0.30
65	0.05	0.30	0.35	0.10	— 0.25
75	0.05	0.45	0.50	0.20	— 0.30
85	0.05	0.60	0.65	0.35	— 0.30
95	0.10	0.85	0.95	0.35	— 0.60
105	0.10	1.10	1.20	0.65	— 0.55
115	0.10	1.50	1.60	1.35	— 0.25
125	0.10	1.85	1.95	2.20	+ 0.25
135	0.15	2.15	2.30	2.65	+ 0.35
145	0.20	2.35	2.55	2.80	+ 0.25
155	0.30	2.35	2.65	2.35	— 0.30
165	0.45	2.15	2.60	2.00	— 0.60
175	0.60	1.85	2.45	1.90	— 0.55
185	0.85	1.50	2.35	2.55	— 0.20
195	1.30	1.10	2.40	3.10	+ 0.70
205	1.95	0.85	2.80	3.20	+ 0.40
215	2.80	0.60	3.40	3.45	+ 0.05
225	3.65	0.45	4.10	4.10	0.00
235	4.50	0.30	4.80	5.35	+ 0.55
245	5.10	0.25	5.35	5.35	0.00
255	5.10	0.20	5.30	5.20	— 0.10
265	4.50	0.15	4.65	4.35	— 0.30
275	3.65	0.10	3.75	3.90	+ 0.15
285	2.80	0.10	2.90	2.65	— 0.25
295	1.95	0.10	2.05	1.65	— 0.40
305	1.30	0.10	1.40	0.80	— 0.60
315	0.85	0.05	0.90	0.45	— 0.45
325	0.60	0.05	0.65	0.45	— 0.20
335	0.45	0.05	0.50	0.55	+ 0.05
345	0.30	0.05	0.35	0.65	+ 0.30
355°	0.20	0.10	0.30	0.80	+ 0.50
totals	44.70	24.50	69.20	69.00	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	250°	1.15	45
Drift II	150°	0.95	24

TABLE VII. REGION VII.

ψ	Calculated numbers			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.30	0.20	0.50	0.90	+ 0.40
15	0.40	0.20	0.60	1.35	+ 0.75
25	0.45	0.20	0.65	1.20	+ 0.55
35	0.55	0.25	0.80	1.20	+ 0.40
45	0.70	0.25	0.95	1.20	+ 0.25
55	0.90	0.30	1.20	1.10	— 0.10
65	1.15	0.35	1.50	1.35	— 0.15
75	1.50	0.40	1.90	1.75	— 0.15
85	1.85	0.50	2.35	3.00	+ 0.65
95	2.25	0.65	2.90	3.45	+ 0.55
105	2.65	0.80	3.45	3.75	+ 0.30
115	2.95	1.05	4.00	3.65	— 0.35
125	3.20	1.30	4.50	4.35	— 0.15
135	3.20	1.65	4.85	5.10	+ 0.25
145	2.95	2.05	5.00	5.20	+ 0.20
155	2.65	2.45	5.10	5.00	— 0.10
165	2.25	2.80	5.05	4.45	— 0.60
175	1.85	3.05	4.90	4.75	— 0.15
185	1.50	3.30	4.80	4.45	— 0.35
195	1.15	3.05	4.20	4.35	+ 0.15
205	0.90	2.80	3.70	3.65	— 0.05
215	0.70	2.45	3.15	3.45	+ 0.30
225	0.55	2.05	2.60	2.55	— 0.05
235	0.45	1.65	2.10	1.75	— 0.35
245	0.40	1.30	1.70	1.00	— 0.70
255	0.30	1.05	1.35	1.10	— 0.25
265	0.30	0.80	1.10	1.10	0.00
275	0.25	0.65	0.90	1.35	+ 0.45
285	0.25	0.50	0.75	0.90	+ 0.15
295	0.20	0.40	0.60	0.65	+ 0.05
305	0.20	0.35	0.55	0.20	— 0.35
315	0.20	0.30	0.50	0.10	— 0.40
325	0.20	0.25	0.45	0.00	— 0.45
335	0.25	0.25	0.50	0.00	— 0.50
345	0.25	0.20	0.45	0.00	— 0.45
355°	0.30	0.20	0.50	0.45	— 0.05
totals	40.10	40.00	80.10	79.80	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	130°	0.75	40
Drift II	185°	0.75	40

TABLE VIII. REGION VIII.

ψ	Calculated numbers			Observed numbers smoothed	$o - c$
	Drift I	Drift II	Total		
5°	0.20	0.05	0.25	0.20	− 0.05
15	0.30	0.05	0.35	0.35	0.00
25	0.50	0.05	0.55	0.20	− 0.35
35	0.75	0.05	0.80	0.55	− 0.25
45	1.15	0.05	1.20	1.10	− 0.10
55	1.85	0.05	1.90	2.10	+ 0.20
65	2.75	0.05	2.80	3.10	+ 0.30
75	3.95	0.10	4.05	4.35	+ 0.30
85	5.25	0.10	5.35	6.00	+ 0.65
95	6.30	0.10	6.40	6.65	+ 0.25
105	7.30	0.15	7.45	7.00	− 0.45
115	6.30	0.20	6.50	5.55	− 0.95
125	5.25	0.30	5.55	4.90	− 0.65
135	3.95	0.45	4.40	3.65	− 0.75
145	2.75	0.65	3.40	3.55	+ 0.15
155	1.85	1.00	2.85	2.90	+ 0.05
165	1.15	1.50	2.65	2.65	0.00
175	0.75	2.15	2.90	2.35	− 0.55
185	0.50	2.80	3.30	2.80	− 0.50
195	0.30	3.45	3.75	3.55	− 0.20
205	0.20	3.90	4.10	4.35	+ 0.25
215	0.15	3.90	4.05	4.35	+ 0.30
225	0.10	3.45	3.55	3.55	0.00
235	0.10	2.80	2.90	2.65	− 0.25
245	0.10	2.15	2.25	1.90	− 0.35
255	0.05	1.50	1.55	1.45	− 0.10
265	0.05	1.00	1.05	1.00	− 0.05
275	0.05	0.65	0.70	0.65	− 0.05
285	0.05	0.45	0.50	0.45	− 0.05
295	0.05	0.30	0.35	0.45	+ 0.10
305	0.05	0.20	0.25	0.80	+ 0.55
315	0.05	0.15	0.20	1.00	+ 0.80
325	0.10	0.10	0.20	1.00	+ 0.80
335	0.10	0.10	0.20	0.55	+ 0.35
345	0.10	0.10	0.20	0.20	0.00
355°	0.15	0.05	0.20	0.10	− 0.10
totals	54.55	34.10	88.65	87.95	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	105°	1.25	54
Drift II	210°	1.15	34

TABLE IX. REGION IX.

ψ	Calculated numbers			Observed numbers smoothed	$o - c$
	Drift I	Drift II	Total		
5°	0.05	0.65	0.70	1.20	+ 0.50
15	0.10	0.60	0.70	1.10	+ 0.40
25	0.10	0.55	0.65	0.65	0.00
35	0.15	0.50	0.65	0.55	− 0.10
45	0.20	0.45	0.65	0.35	− 0.30
55	0.25	0.40	0.65	0.55	− 0.10
65	0.45	0.40	0.85	1.00	+ 0.15
75	0.55	0.35	0.90	1.35	+ 0.45
85	0.85	0.35	1.20	1.45	+ 0.25
95	1.25	0.35	1.60	1.35	− 0.25
105	1.85	0.35	2.20	1.65	− 0.55
115	2.55	0.35	2.90	2.45	− 0.45
125	3.25	0.35	3.60	3.55	− 0.05
135	3.80	0.40	4.20	4.45	+ 0.25
145	4.05	0.40	4.45	4.75	+ 0.30
155	3.80	0.45	4.25	4.20	− 0.05
165	3.25	0.50	3.75	3.65	− 0.10
175	2.55	0.55	3.10	3.00	− 0.10
185	1.85	0.60	2.45	2.90	+ 0.45
195	1.25	0.65	1.90	2.20	+ 0.30
205	0.85	0.75	1.60	1.55	− 0.05
215	0.55	0.80	1.35	1.10	− 0.25
225	0.45	0.90	1.35	1.00	− 0.35
235	0.25	1.00	1.25	1.10	− 0.15
245	0.20	1.05	1.25	0.90	− 0.35
255	0.15	1.10	1.25	1.00	− 0.25
265	0.10	1.15	1.25	1.20	− 0.05
275	0.10	1.20	1.30	1.55	+ 0.25
285	0.05	1.20	1.25	1.45	+ 0.20
295	0.05	1.15	1.20	1.10	− 0.10
305	0.05	1.10	1.15	0.75	− 0.40
315	0.05	1.05	1.10	0.65	− 0.45
325	0.05	1.00	1.05	0.65	− 0.40
335	0.05	0.90	0.95	0.90	− 0.05
345	0.05	0.80	0.85	1.10	+ 0.25
355°	0.05	0.75	0.80	1.45	+ 0.65
totals	35.20	25.10	60.30	59.80	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	145°	1.15	35
Drift II	280°	0.35	25

TABLE X. REGION X.

ψ	Calculated numbers			Observed numbers smoothed	$o-c$
	Drift I	Drift II	Total		
5°	0.05	0.20	0.25	0.10	— 0.15
15	0.05	0.25	0.30	0.20	— 0.10
25	0.05	0.30	0.35	0.45	+ 0.10
35	0.05	0.35	0.40	0.45	+ 0.05
45	0.05	0.40	0.45	0.55	+ 0.10
55	0.05	0.45	0.50	0.55	+ 0.05
65	0.05	0.55	0.60	0.65	+ 0.05
75	0.05	0.65	0.70	0.65	— 0.05
85	0.05	0.75	0.80	0.55	— 0.25
95	0.05	0.90	0.95	0.65	— 0.30
105	0.10	1.00	1.10	0.75	— 0.35
115	0.10	1.10	1.20	1.20	0.00
125	0.20	1.20	1.40	1.55	+ 0.15
135	0.30	1.25	1.55	1.90	+ 0.35
145	0.50	1.25	1.75	1.90	+ 0.15
155	0.80	1.20	2.00	1.75	— 0.25
165	1.45	1.10	2.55	2.00	— 0.55
175	2.50	1.00	3.50	3.00	— 0.50
185	3.90	0.90	4.80	4.75	— 0.05
195	5.35	0.75	6.10	6.75	+ 0.65
205	6.90	0.65	7.55	8.00	+ 0.45
215	7.60	0.55	8.15	7.65	— 0.50
225	6.90	0.45	7.35	6.55	— 0.80
235	5.35	0.40	5.75	4.90	— 0.85
245	3.90	0.35	4.25	4.20	— 0.05
255	2.50	0.30	2.80	3.20	+ 0.40
265	1.45	0.25	1.70	2.90	+ 1.20
275	0.80	0.20	1.00	1.75	+ 0.75
285	0.50	0.20	0.70	1.10	+ 0.40
295	0.30	0.20	0.50	0.45	— 0.05
305	0.20	0.20	0.40	0.35	— 0.05
315	0.10	0.20	0.30	0.20	— 0.10
325	0.10	0.20	0.30	0.10	— 0.20
335	0.05	0.20	0.25	0.00	— 0.25
345	0.05	0.20	0.25	0.00	— 0.25
355°	0.05	0.20	0.25	0.00	— 0.25
totals	52.45	20.30	72.75	71.70	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	215°	1.45	52
Drift II	140°	0.55	20

TABLE XI. REGION XI.

ψ	Calculated numbers.			Observed numbers smoothed	$o-c$
	Drift I	Drift II	Total		
5°	0.95	0.00	0.95	0.90	— 0.05
15	0.80	0.00	0.80	1.10	+ 0.30
25	0.70	0.00	0.70	1.10	+ 0.40
35	0.60	0.00	0.60	1.00	+ 0.40
45	0.55	0.00	0.55	0.65	+ 0.10
55	0.45	0.00	0.45	0.45	0.00
65	0.45	0.00	0.45	0.45	0.00
75	0.40	0.00	0.40	0.65	+ 0.25
85	0.40	0.00	0.40	0.75	+ 0.35
95	0.40	0.00	0.40	0.65	+ 0.25
105	0.40	0.05	0.45	0.35	— 0.10
115	0.40	0.10	0.50	0.20	— 0.30
125	0.45	0.15	0.60	0.55	— 0.05
135	0.45	0.35	0.80	1.00	+ 0.20
145	0.55	0.75	1.30	1.55	+ 0.25
155	0.60	1.35	1.95	1.75	— 0.20
165	0.70	2.10	2.80	2.75	— 0.05
175	0.80	2.65	3.45	3.35	— 0.10
185	0.95	2.65	3.60	3.65	+ 0.05
195	1.10	2.10	3.20	2.55	— 0.65
205	1.30	1.35	2.65	1.65	— 1.00
215	1.55	0.75	2.30	1.20	— 1.10
225	1.80	0.35	2.15	1.75	— 0.40
235	2.15	0.15	2.30	2.45	+ 0.15
245	2.35	0.10	2.45	3.10	+ 0.65
255	2.60	0.05	2.65	3.20	+ 0.55
265	2.75	0.00	2.75	3.20	+ 0.45
275	2.80	0.00	2.80	2.75	— 0.05
285	2.75	0.00	2.75	2.55	— 0.20
295	2.60	0.00	2.60	2.20	— 0.40
305	2.35	0.00	2.35	2.10	— 0.25
315	2.15	0.00	2.15	2.00	— 0.15
325	1.80	0.00	1.80	2.00	+ 0.20
335	1.55	0.00	1.55	1.75	+ 0.20
345	1.30	0.00	1.30	1.35	+ 0.05
355°	1.10	0.00	1.10	1.00	— 0.10
totals	45.00	15.00	60.00	59.65	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	275°	0.55	45
Drift II	180°	1.85	15

TABLE XII. REGION XIII.

ψ	Calculated numbers			Observed numbers smoothed	$o-c$
	Drift I	Drift II	Total		
5°	0.15	1.60	1.75	1.55	—0.20
15	0.25	1.45	1.70	1.45	—0.25
25	0.55	1.30	1.85	1.55	—0.30
35	1.15	1.15	2.30	2.35	+0.05
45	2.20	1.05	3.25	3.35	+0.10
55	3.85	0.95	4.80	4.90	+0.10
65	5.65	0.90	6.55	6.55	0.00
75	7.05	0.80	7.85	7.75	—0.10
85	7.05	0.75	7.80	7.65	—0.15
95	5.65	0.70	6.35	5.90	—0.45
105	3.85	0.65	4.50	3.75	—0.75
115	2.20	0.65	2.85	2.45	—0.40
125	1.15	0.65	1.80	2.10	+0.30
135	0.55	0.65	1.20	1.75	+0.55
145	0.25	0.65	0.90	1.10	+0.20
155	0.15	0.65	0.80	0.75	—0.05
165	0.10	0.70	0.80	0.90	+0.10
175	0.05	0.75	0.80	1.20	+0.40
185	0.05	0.80	0.85	1.45	+0.60
195	0.00	0.90	0.90	1.45	+0.55
205	0.00	0.95	0.95	1.55	+0.60
215	0.00	1.05	1.05	1.35	+0.30
225	0.00	1.15	1.15	1.35	+0.20
235	0.00	1.30	1.30	1.10	—0.20
245	0.00	1.45	1.45	1.20	—0.25
255	0.00	1.60	1.60	1.55	—0.05
265	0.00	1.75	1.75	2.00	+0.25
275	0.00	1.90	1.90	2.20	+0.30
285	0.00	2.00	2.00	1.75	—0.25
295	0.00	2.10	2.10	1.35	—0.75
305	0.00	2.15	2.15	1.20	—0.95
315	0.00	2.15	2.15	1.55	—0.60
325	0.00	2.10	2.10	2.00	—0.10
335	0.05	2.00	2.05	2.35	+0.30
345	0.05	1.90	1.95	2.35	+0.40
355°	0.10	1.80	1.90	2.10	+0.20
totals	42.10	45.05	87.15	86.85	

Results:

	direction ψ_0	velocity $hV \sin\lambda$	stars N
Drift I	100°	1.75	42
Drift II	230°	0.35	45

TABLE XIII. REGION XIV.

ψ	Calculated numbers			Observed numbers smoothed	$o-c$
	Drift I	Drift II	Total		
5°	1.55	0.50	2.05	1.35	—0.70
15	2.30	0.45	2.75	2.45	—0.30
25	3.10	0.50	3.60	3.45	—0.15
35	3.65	0.50	4.15	4.45	+0.30
45	3.65	0.50	4.15	3.90	—0.25
55	3.10	0.55	3.65	3.45	—0.20
65	2.30	0.65	2.95	2.65	—0.30
75	1.55	0.70	2.25	2.75	+0.50
85	0.95	0.80	1.75	2.45	+0.70
95	0.55	0.95	1.50	2.45	+0.95
105	0.30	1.10	1.40	2.00	+0.60
115	0.20	1.30	1.50	2.00	+0.50
125	0.10	1.55	1.65	1.90	+0.25
135	0.05	1.85	1.90	1.90	0.00
145	0.05	2.20	2.25	2.20	—0.05
155	0.05	2.55	2.60	2.65	+0.05
165	0.05	2.85	2.90	3.10	+0.20
175	0.00	3.10	3.10	3.00	—0.10
185	0.00	3.30	3.30	2.75	—0.55
195	0.00	3.40	3.40	3.00	—0.40
205	0.00	3.30	3.30	3.20	—0.10
215	0.00	3.10	3.10	3.45	+0.35
225	0.00	2.85	2.85	3.00	+0.15
235	0.00	2.55	2.55	2.35	—0.20
245	0.00	2.20	2.20	1.55	—0.65
255	0.00	1.85	1.85	1.35	—0.50
265	0.00	1.55	1.55	1.45	—0.10
275	0.05	1.30	1.35	1.55	+0.20
285	0.05	1.10	1.15	1.10	—0.05
295	0.05	0.95	1.00	1.00	0.00
305	0.05	0.80	0.85	1.00	+0.15
315	0.10	0.70	0.80	1.45	+0.65
325	0.20	0.65	0.85	1.35	+0.50
335	0.30	0.55	0.85	1.20	+0.35
345	0.55	0.50	1.05	1.00	—0.05
355°	0.95	0.50	1.45	1.10	—0.35
totals	25.80	53.75	79.55	80.95	

Results:

	direction ψ_0	velocity $hV \sin\lambda$	stars N
Drift I	40°	1.45	26
Drift II	195°	0.55	54

TABLE XIV. REGIONS XV AND XII.

ψ	Calculated numbers			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.95	1.05	2.00	3.00	+ 1.00
15	0.60	1.10	1.70	2.45	+ 0.75
25	0.40	1.10	1.50	2.10	+ 0.60
35	0.30	1.20	1.50	2.10	+ 0.60
45	0.20	1.30	1.50	1.80	+ 0.30
55	0.15	1.50	1.65	1.75	+ 0.10
65	0.15	1.70	1.85	1.65	— 0.20
75	0.10	1.95	2.05	2.00	— 0.05
85	0.10	2.30	2.40	2.20	— 0.20
95	0.10	2.65	2.75	2.55	— 0.20
105	0.10	3.20	3.30	3.10	— 0.20
115	0.10	3.80	3.90	4.00	+ 0.10
125	0.10	4.50	4.60	5.25	+ 0.65
135	0.10	5.20	5.30	6.00	+ 0.70
145	0.10	6.00	6.10	6.55	+ 0.45
155	0.15	6.75	6.90	6.75	— 0.15
165	0.15	7.20	7.35	6.55	— 0.80
175	0.20	7.45	7.65	5.80	— 1.85
185	0.30	7.45	7.75	4.95	— 2.80
195	0.40	7.20	7.60	6.00	— 1.60
205	0.60	6.75	7.35	7.70	+ 0.35
215	0.95	6.00	6.95	9.55	+ 2.60
225	1.45	5.20	6.65	8.75	+ 2.10
235	2.30	4.50	6.80	7.75	+ 0.95
245	3.65	3.80	7.45	6.10	— 1.35
255	5.20	3.20	8.40	6.55	— 1.85
265	7.45	2.65	10.10	8.20	— 1.90
275	9.35	2.30	11.65	10.85	— 0.80
285	10.55	1.95	12.50	12.75	+ 0.25
295	10.55	1.70	12.25	12.65	+ 0.40
305	9.35	1.50	10.85	11.35	+ 0.50
315	7.45	1.30	8.75	8.65	— 0.10
325	5.20	1.20	6.40	6.20	— 0.20
335	3.65	1.10	4.75	4.40	— 0.35
345	2.30	1.10	3.40	3.90	+ 0.50
355°	1.45	1.05	2.50	3.65	+ 1.15
totals	86.20	119.90	206.10	205.55	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	290°	1.25	86
Drift II	180°	0.55	120

TABLE XV. REGION XVI.

ψ	Calculated numbers.			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.05	0.10	0.15	0.00	— 0.15
15	0.05	0.10	0.15	0.00	— 0.15
25	0.05	0.15	0.20	0.00	— 0.20
35	0.05	0.15	0.20	0.00	— 0.20
45	0.05	0.20	0.25	0.10	— 0.15
55	0.05	0.25	0.30	0.20	— 0.10
65	0.05	0.30	0.35	0.35	0.00
75	0.05	0.40	0.45	0.45	0.00
85	0.05	0.50	0.55	0.75	+ 0.20
95	0.05	0.65	0.70	1.20	+ 0.50
105	0.10	0.80	0.90	1.45	+ 0.55
115	0.10	0.95	1.05	1.45	+ 0.40
125	0.15	1.05	1.20	1.20	0.00
135	0.20	1.10	1.30	1.20	— 0.10
145	0.30	1.10	1.40	1.10	— 0.30
155	0.50	1.05	1.55	1.20	— 0.35
165	0.80	0.95	1.75	1.20	— 0.55
175	1.30	0.80	2.10	1.90	— 0.20
185	2.15	0.65	2.80	2.55	— 0.25
195	3.40	0.50	3.90	3.65	— 0.25
205	4.90	0.40	5.30	5.10	— 0.20
215	6.30	0.30	6.60	6.55	— 0.05
225	7.15	0.25	7.40	8.00	+ 0.60
235	7.15	0.20	7.35	7.20	— 0.15
245	6.30	0.15	6.45	6.55	+ 0.10
255	4.90	0.15	5.05	4.55	— 0.50
265	3.40	0.10	3.50	3.90	— 0.40
275	2.15	0.10	2.25	2.55	+ 0.30
285	1.30	0.10	1.40	2.00	+ 0.60
295	0.80	0.10	0.90	1.20	+ 0.30
305	0.50	0.10	0.60	0.75	+ 0.15
315	0.30	0.05	0.35	0.35	0.00
325	0.20	0.05	0.25	0.10	— 0.15
335	0.15	0.10	0.25	0.00	— 0.25
345	0.10	0.10	0.20	0.00	— 0.20
355°	0.10	0.10	0.20	0.00	— 0.20
totals	55.20	14.10	69.30	68.75	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	230°	1.35	55
Drift II	140°	0.75	14

TABLE XVI. REGION XVII.

ψ	Calculated numbers			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.15	0.55	0.70	0.45	— 0.25
15	0.20	0.55	0.75	0.65	— 0.10
25	0.25	0.60	0.85	1.00	+ 0.15
35	0.25	0.60	0.85	1.10	+ 0.25
45	0.35	0.65	1.00	1.20	+ 0.20
55	0.45	0.65	1.10	1.45	+ 0.35
65	0.55	0.75	1.30	2.10	+ 0.80
75	0.75	0.80	1.55	2.45	+ 0.90
85	1.05	0.85	1.90	2.65	+ 0.75
95	1.45	0.95	2.40	2.55	+ 0.15
105	2.05	1.05	3.10	3.10	0.00
115	2.70	1.20	3.90	3.75	— 0.15
125	3.50	1.30	4.80	4.65	— 0.15
135	4.30	1.45	5.75	5.35	— 0.40
145	4.75	1.60	6.35	5.90	— 0.45
155	5.00	1.70	6.70	6.55	— 0.15
165	4.75	1.85	6.60	6.75	+ 0.15
175	4.30	1.90	6.20	6.45	+ 0.25
185	3.50	1.95	5.45	5.35	— 0.10
195	2.70	1.95	4.65	4.10	— 0.55
205	2.05	1.90	3.95	3.10	— 0.85
215	1.45	1.85	3.30	2.55	— 0.75
225	1.05	1.70	2.75	2.45	— 0.30
235	0.75	1.60	2.35	2.45	+ 0.10
245	0.55	1.45	2.00	2.20	+ 0.20
255	0.45	1.30	1.75	1.90	+ 0.15
265	0.35	1.20	1.55	1.75	+ 0.20
275	0.25	1.05	1.30	1.90	+ 0.60
285	0.25	0.95	1.20	1.65	+ 0.45
295	0.20	0.85	1.05	1.20	+ 0.15
305	0.15	0.80	0.95	0.65	— 0.30
315	0.15	0.75	0.90	0.45	— 0.45
325	0.15	0.65	0.80	0.35	— 0.45
335	0.15	0.65	0.80	0.20	— 0.60
345	0.15	0.60	0.75	0.20	— 0.55
355°	0.15	0.60	0.75	0.20	— 0.55
totals	51.25	40.80	92.05	90.75	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	155°	0.95	51
Drift II	190°	0.35	41

TABLE XVII. REGION XVIII.

ψ	Calculated numbers			Observed numbers smoothed	o — c
	Drift I	Drift II	Total		
5°	0.95	0.50	1.45	2.20	+ 0.75
15	1.35	0.45	1.80	2.75	+ 0.95
25	1.85	0.40	2.25	3.10	+ 0.85
35	2.50	0.35	2.85	3.45	+ 0.60
45	3.40	0.35	3.75	3.65	— 0.10
55	4.50	0.30	4.80	4.35	— 0.45
65	5.65	0.30	5.95	5.35	— 0.60
75	6.50	0.30	6.80	6.55	— 0.25
85	7.15	0.30	7.45	7.35	— 0.10
95	7.15	0.30	7.45	7.45	0.00
105	6.50	0.30	6.80	7.00	+ 0.20
115	5.65	0.35	6.00	5.90	— 0.10
125	4.50	0.35	4.85	4.75	— 0.10
135	3.40	0.40	3.80	3.55	— 0.25
145	2.50	0.45	2.95	2.85	— 0.10
155	1.85	0.50	2.35	2.20	— 0.15
165	1.35	0.60	1.95	1.65	— 0.30
175	0.95	0.65	1.60	1.45	— 0.15
185	0.75	0.75	1.50	1.55	+ 0.05
195	0.55	0.85	1.40	1.75	+ 0.35
205	0.45	1.00	1.45	1.75	+ 0.30
215	0.35	1.10	1.45	1.55	+ 0.10
225	0.30	1.25	1.55	1.75	+ 0.20
235	0.25	1.35	1.60	1.90	+ 0.30
245	0.25	1.45	1.70	2.00	+ 0.30
255	0.20	1.50	1.70	1.45	— 0.25
265	0.20	1.50	1.70	0.90	— 0.80
275	0.20	1.45	1.65	0.55	— 1.10
285	0.20	1.35	1.55	0.65	— 0.90
295	0.25	1.25	1.50	1.10	— 0.40
305	0.25	1.10	1.35	1.45	+ 0.10
315	0.30	1.00	1.30	1.55	+ 0.25
325	0.35	0.85	1.20	1.35	+ 0.15
335	0.45	0.75	1.20	1.35	+ 0.15
345	0.55	0.65	1.20	1.20	0.00
355°	0.75	0.60	1.35	1.45	+ 0.10
totals	74.30	26.90	101.20	100.80	

Results:

	direction ψ_0	velocity $hV \sin \lambda$	stars N
Drift I	90°	0.95	74
Drift II	260°	0.45	27

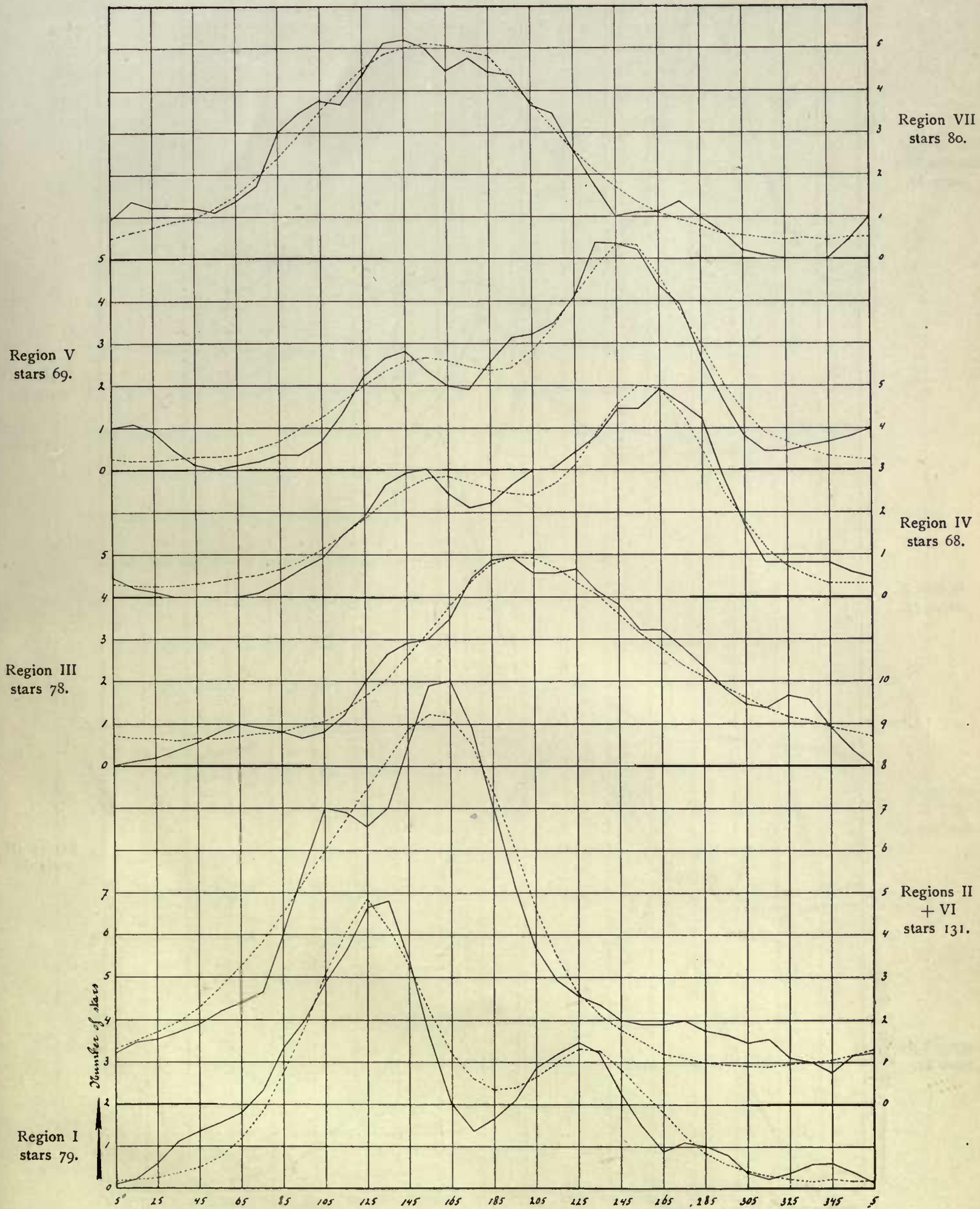


Fig. III. Comparison of observation and theory.

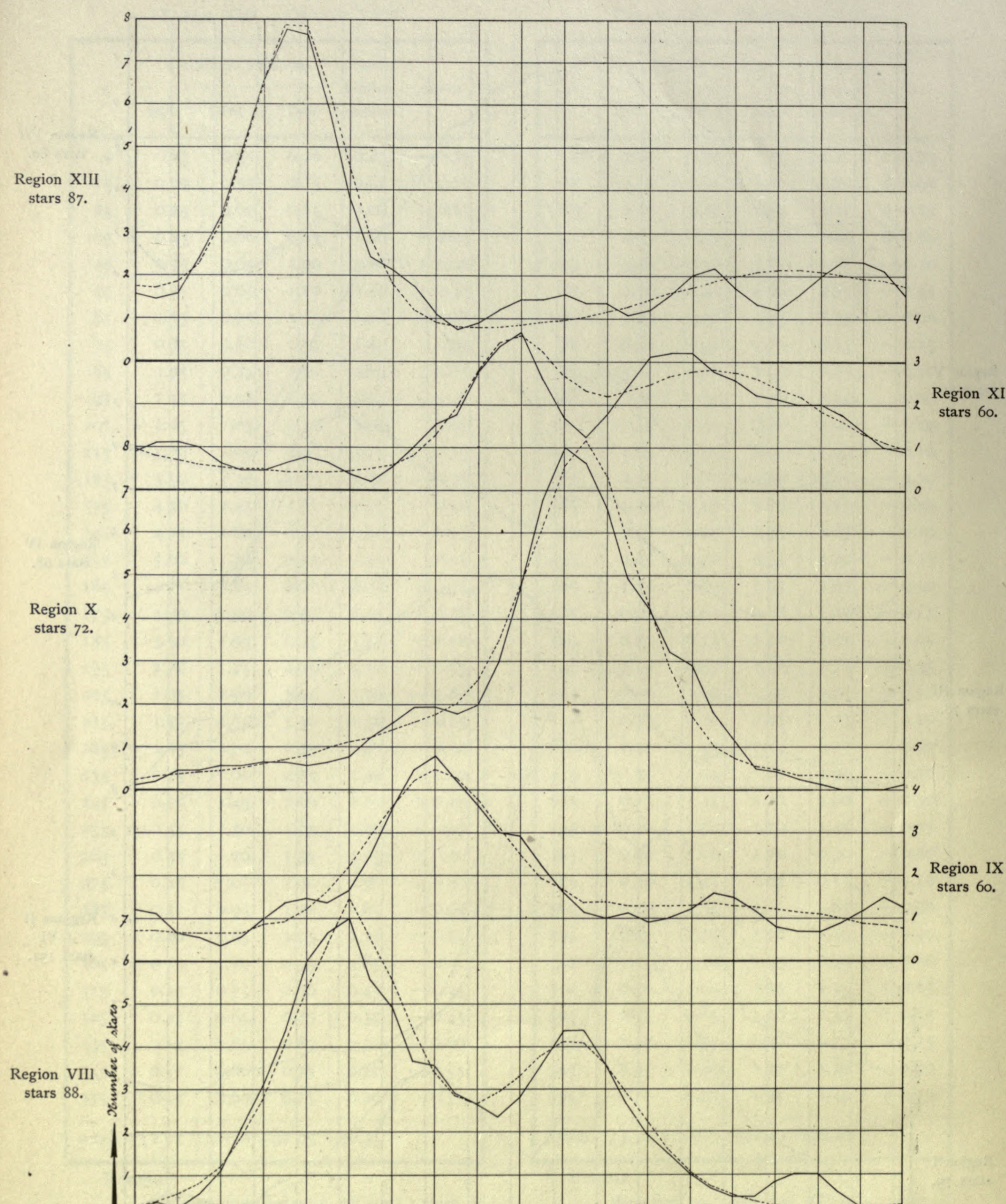


Fig. IV. Comparison of observation and theory.

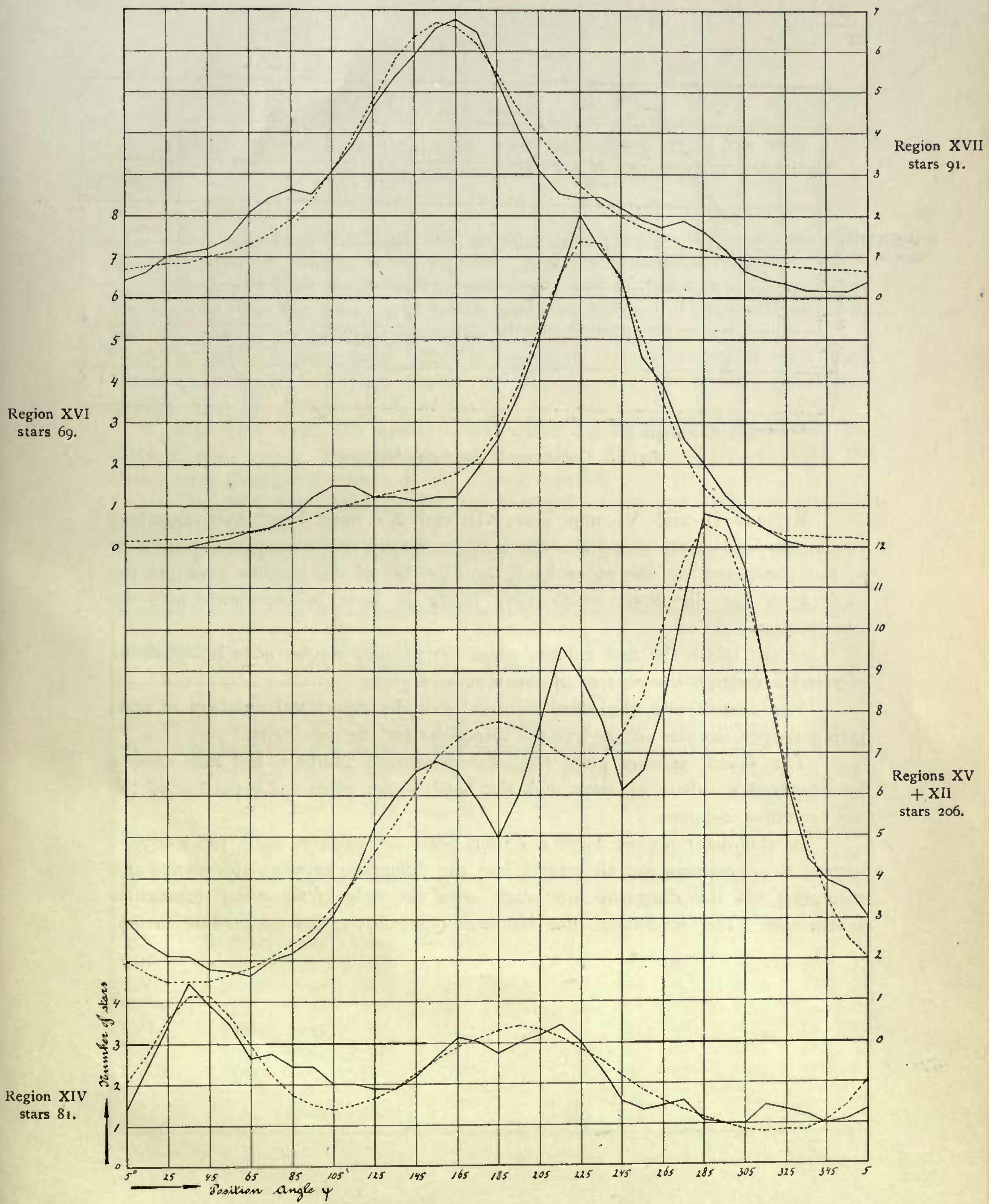
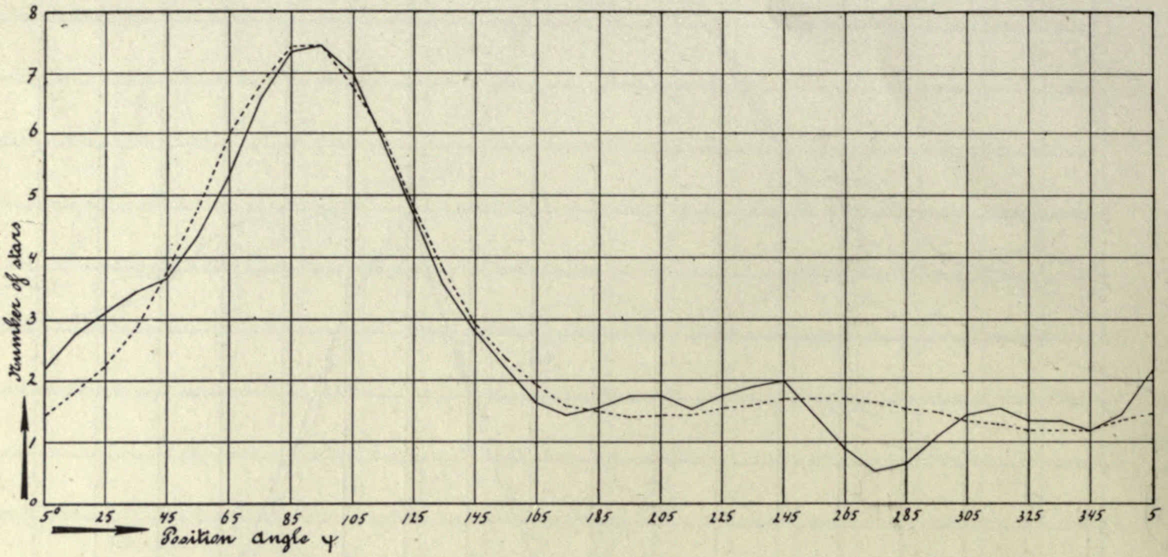


Fig. V. Comparison of observation and theory.



Region XVIII
stars 101.

Fig. VI. Comparison of observation and theory.

Regions II and VI and also XII and XV have been taken together, because on one hand the difference between theory and observation proved to be too great and on the other hand the direction of the maxima diverged too much from the directions which they ought to have in accordance with the other regions.

In the tables the first column gives the position angles ψ inclining to the hourcircles through the centres of the various regions.

The second and the third column give the theoretical numbers of stars having proper motions in the various directions for the two drifts.

The fourth column gives the total theoretical numbers, the fifth column the observed numbers of stars and the last column gives a comparison of the two preceding columns.

In the diagrams we have not used polar co-ordinates as in the analysis, because it is easier to get an insight into the difference between observation and calculation, if the diagrams are made with the help of the usual rectangular co-ordinates. The continuous line indicates observation, the dotted line theory.

CHAPTER V.

RESULTS OF THE ANALYSIS AND DETERMINATION OF THE ANTIVERTICES OF THE TWO DRIFTS.

a. Results.

In the preceding chapter we found for each region the constants of the two single curves, which, taken together, give the theoretical curves.

Each of these single curves represents one of the two drifts, to which we suppose that the stars, used in the analysis, belong. The directions of the maxima indicate the apparent directions of the drifts for that region.

In order to determine which of the single curves belongs to drift I and which to drift II, we have drawn the directions of the maxima on a globe starting from the centres of the regions.

For each drift the great circles which we now obtain will nearly pass through one point, the antivertex of the drift. The directions, of which the circles pass through the same point, belong together.

The following table shows the constants of the two drifts separately in the several regions.

The position angle ψ_0 give the apparent direction of the two drifts.

TABLE XVIII.

Regions	Centres		total numbers of stars	Drift I			weights	Drift II			weights
	R. A.	Decl.		ψ_0	$hV \sin \lambda$	N		ψ_0	$hV \sin \lambda$	N	
I	19°	+ 1°	79	125°	1.25	52	6	230°	1.15	27	4
II + VI	69°	+ 1°	131	130°	0.55	100	9	170°	1.35	32	5
III	111°	— 1°	78	240°	0.50	40	6	190°	0.85	37	5
IV	156°	0	68	260°	1.25	38	12	160°	0.85	30	11
V	203°	0	69	250°	1.15	45	9	150°	0.95	24	8
VI	246°	+ 1°	(72)	see II							
VII	287°	— 3°	80	130°	0.75	40	5	185°	0.75	40	12
VIII	338°	— 3°	88	105°	1.25	54	7	210°	1.15	34	6
IX	45°	+ 45°	60	145°	1.15	35	6	280°	0.35	25	5
X	132°	+ 42°	72	215°	1.45	52	4	140°	0.55	20	8
XI	223° 30′	+ 41°	60	275°	0.55	45	4	180°	1.85	15	4
XII	313° 30′	+ 44°	(107)	see XV							
XIII	North Pole		87	100°	1.75	42	5	230°	0.35	45	7
XIV	46° 30′	— 42°	81	40°	1.45	26	12	195°	0.55	54	7
XV + XII	135°	— 45°	206	290°	1.25	86	11	180°	0.55	120	7
XVI	228°	— 43°	69	230°	1.35	55	6	140°	0.75	14	7
XVII	312°	— 41°	91	155°	0.95	51	4	190°	0.35	41	5
XVIII	South Pole		101	90°	0.95	74	9	260°	0.45	27	8
		totals	1420			835				585	

With the aid of the data in this table, we can calculate the different constants of the two drifts

b. The antivertices of the two drifts.

In the first place we determine the antivertices of the two drifts, as by mean of these antivertices we wish to make an improvement on the distribution of the stars over the two drifts and the values of $h V \sin \lambda$ in the different regions.

To determine the antivertices, we draw the great circles in the directions ψ_0 on the globe as accurately as possible by measuring off ψ_0 on other great circles, whose poles lie in the centres of the regions. As for each drift these circles do not pass through one point, we try to find a point to which all the circles pass as close as possible

To find such a point, we take the circles in small groups together and replace each group by a point or a line.

Then we combine these points and lines into the desired point. If we do this in several ways, we obtain several determinations of the antivertices and these together give us a satisfactory determination of these points themselves, viz:

Drift I R. A. 90°. Decl. — 18°.
Drift II. R. A. 268°. Decl. — 65°.

As it is difficult to find in this manner a probable error for the antivertices, we have made a calculation which procures us a slight correction and at the same time a probable error for the determination of the antivertices. By means of the distances from the great circles to the points which we have found on the globe for the antivertices of drift I and drift II, we have computed for each drift a point situated so, that the sum of the squares of the distances from the circles to this point is a minimum. The weights for the two drifts in each region have been taken in agreement with the numbers of stars, the difference between the observed and the theoretical numbers and with the greater or smaller deviation from the preliminary antivertices and the mean values of $h V$.

The weights which we have used are given in Table XVIII.

The antivertices corrected in this way and their probable errors are:

Apparent antivertex of drift I
R. A. — 87°1 ± 2°.5 Decl. — 15°9 ± 2°.3.
Apparent antivertex of drift II
R. A. 265°5 ± 3°.4 Decl. — 65°0 ± 3°.3.

Of course it is possible to determine these points only by means of calculation, but this is circumstantial and the accuracy of the data is not in accordance

The great circles near the antivertices in a stereographical projection.

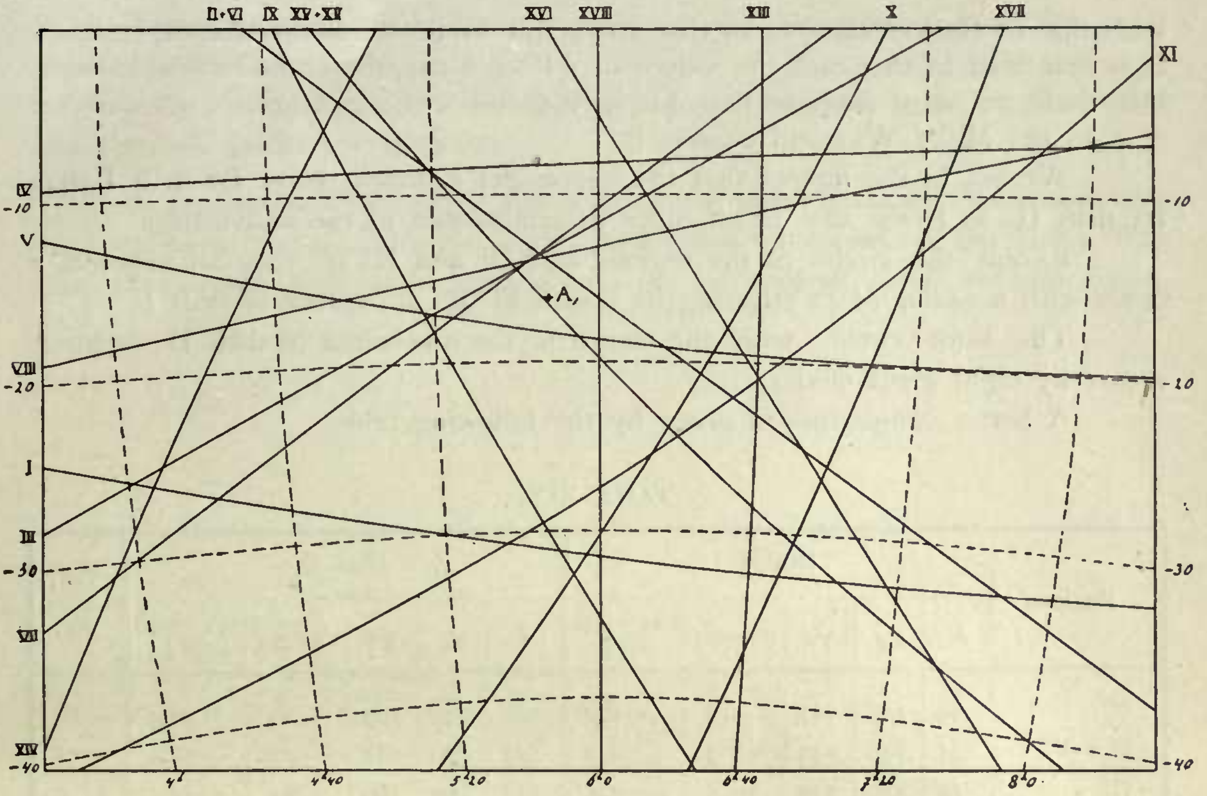


Fig. VII. Antivertex of drift I.

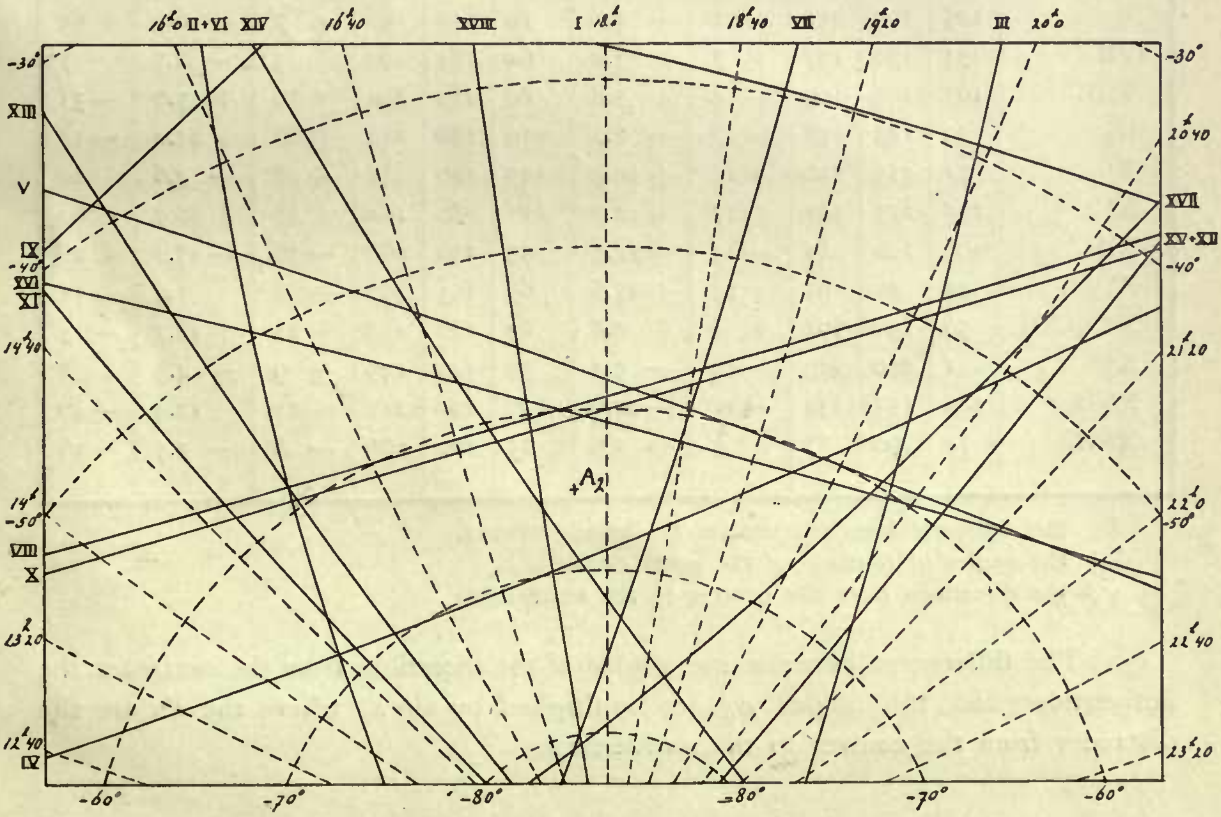


Fig. VIII. Antivertex of drift II.

with that of the calculation, so that the result would not answer the expectation. It is true that in this case the values of $hV \sin \lambda$ can also be taken into account, but then we must suppose that hV only varies with $\sin \lambda$ and is not different e. g. in the Milky Way and outside it.

We see in the figures that the agreement is much better for drift I than for drift II, as is the case in all other determinations of the antivertices.

Except the circles of the regions I, XIV and XVII, they all intersect a circle with a radius of 12°, having its centre in the antivertex of drift I.

The same circle, with the centre in the antivertex of drift II, is intersected by eight great circles.

A better comparison is given by the following table.

Table XIX.

Regions	Drift I					Drift II					Gal. lat.
	λ	ψ_0	ψ_0'	$\psi_0-\psi_0'$	$(\psi_0-\psi_0') \sin \lambda$	λ	ψ_0	ψ_0'	$\psi_0-\psi_0'$	$(\psi_0-\psi_0') \sin \lambda$	
I	68°	125°	107°	−18°	−16.7°	101°	230°	203°	+27°	+26.5°	−62°
II + VI	25	130	133	+ 3	+ 1.3	115	170	188	−18	−16.4	−28
III	29	240	235	− 5	− 2.4	111	190	169	+21	+19.5	+ 9
IV	70	260	253	− 7	− 6.6	98	160	156	+ 4	+ 4.0	+48
V	115	250	252	+ 2	+ 1.8	79	150	157	− 7	− 6.9	+61
VII	151	130	137	+ 7	+ 3.4	64	185	190	− 5	− 4.5	− 7
VIII	107	105	109	+ 4	+ 3.8	80	210	204	+ 6	+ 5.9	−51
IX	73	145	138	− 7	− 6.7	150	280	214	+66	+33.0	−12
X	71	215	226	+11	+10.5	145	140	148	− 8	− 4.6	+40
XI	135	275	292	+17	+12.1	111	180	162	+18	+16.7	+60
XIII	106	100	87	−13	−12.5	155	230	266	−36	−15.1	+26
XIV	43	40	65	+25	+15.6	68	195	197	− 2	− 1.9	−55
XV + XII	50	290	291	+ 1	+ 0.8	63	180	159	+21	+18.7	− 3
XVI	111	230	220	−10	− 9.3	30	140	149	− 9	− 4.5	+ 8
XVII	109	155	134	−21	−20.0	36	190	211	−21	−12.4	−40
XVIII	74	90	87	− 3	− 2.9	25	260	266	− 6	− 2.5	−26

λ the distances from the centres to the antivertices.
ψ_0 the angles of position of the great circles.
ψ_0' the directions from the centres to the antivertices.

The differences between the angles of the directions from the centres to the antivertices and the angles ψ_0 are multiplied by $\sin \lambda$, where the λ's are the distances from the centres to the antivertices.

These products give the distances of the antivertices from the great circles in the directions ψ_0.

A greater accuracy could hardly be expected, seeing the small number of stars used in the investigation.

A systematic deviation of the great circles from the determined antivertices has not been found. Nor do the regions which lie nearest to the Milky Way give antivertices that differ greatly from the antivertices which we have found.

CHAPTER VI.

FURTHER RESULTS OF THE ANALYSIS.

a. Correction of the values of $hV \sin \lambda$ and N.

Before we derive the speed of the two drifts and the distribution of the stars over the two drifts from the values of $hV \sin \lambda$ and N given in table XVIII, we shall first correct these values with the aid of the directions of the two drifts which we found in the preceding chapter. That it is necessary to make this correction, will appear from the following description.

As we have seen, the antivertices of the two drifts have been derived from the directions of the maxima of the single-drift curves out of which the theoretical curves in each region were composed.

In the different regions these single-drift curves do not lie in the proper direction, i. e. the direction towards the antivertices. The last table shows that they all deviate more or less from this direction. So we shall have to shift these curves if we wish them to coincide with the directions of the two drifts. But when we compose them in this new position into the theoretical curve, they will represent the observed curve very inaccurately. Therefore we have thought it necessary to modify the single-drift curves in such a manner that, while having a direction of the maxima towards the antivertices, they form a compound curve which agrees as accurately as possible with the observed curve. This modification can be brought about by varying the constants $hV \sin \lambda$ and N.

So we now have to solve the following problem. Given the observed curves and moreover the directions towards the antivertices; to be derived for each region out of two single-drift curves having their maxima in the direction towards the antivertices, a compound curve which corresponds as closely as possible with the given curves.

It proved to be very difficult to find a satisfactory agreement now that the position angles ψ were given. For several regions we did not succeed in finding a better agreement. For 7 regions we have thought it advisable to give somewhat altered values of $hV \sin \lambda$ and N.

Although even with the altered values of $hV \sin \lambda$ and N the agreement is not so good as at first, when also ψ could be varied, yet it is better than taking a curve with the original $hV \sin \lambda$ and N with a ψ giving the directions to the antivertices.

After these corrections we have now got the following data for the derivation of the other constants of the two drifts.

TABLE XX.

Regions	Centres		total numbers of stars	Drift I			weights	Drift II			weights
	R. A.	Decl.		ψ_0	$hV \sin \lambda$	N		ψ_0	$hV \sin \lambda$	N	
I	19°	+ 1°	79	125°	1.25	52	6	230°	1.15	27	4
II + VI	69°	+ 1°	131	130°	0.55	99	9	170°	1.35	32	5
III	111°	— 1°	78	240°	0.65	43	6	190°	0.65	35	5
IV	156°	0	68	260°	1.25	38	12	160°	0.85	30	11
V	203°	0	69	250°	1.15	45	9	150°	0.95	24	8
VI	246°	+ 1°	(72)	see II							
VII	287°	— 3°	80	130°	0.75	55	5	185°	0.85	25	12
VIII	338°	— 3°	88	105°	1.25	54	7	210°	1.15	34	6
IX	45°	+ 45°	60	145°	1.35	30	6	280°	0.35	30	5
X	132°	+ 42°	72	215°	1.45	52	4	140°	0.55	20	8
XI	223° 30′	+ 41°	60	275°	0.65	40	4	180°	1.55	20	4
XII	313° 30′	+ 44°	(107)	see XV							
XIII	North Pole		87	100°	1.75	42	5	230°	0.35	45	7
XIV	46° 30′	— 42°	81	40°	0.95	21	12	195°	0.35	60	7
XV + XII	135°	— 45°	206	290°	1.25	86	11	180°	0.35	120	7
XVI	228°	— 43°	69	230°	1.35	55	6	140°	0.35	14	7
XVII	312°	— 41°	91	155°	0.95	50	4	190°	0.35	41	5
XVIII	South Pole		101	90°	0.95	74	9	260_0	0.45	27	8
		totals	1420			836				584	

This table is the same as Table XVIII, except for regions III, VII, IX, XI, XIV, XV, XVI in the columns headed $hV \sin \lambda$ and N.

b. Derivation of the speed of the star-drifts.

In the mathematical theory we have supposed that the quantity hV has a constant value for each drift. So, if we divide the value of $hV \sin \lambda$ which we have found in the different regions, by the value of $\sin \lambda$ of the same region, we must obtain the same value for hV. As a matter of fact, we find different values of hV in each region. From these values we have computed with least squares the mean value of hV for each drift.

So we find for

Drift I $hV_1 = 1{,}35$ prob. err. 0,11.
Drift II $hV_2 = 0{,}91$ „ „ 0,16.

Table XXI shows that the disagreement for drift I is not very great The disagreement for drift II is much greater.

TABLE XXI.

Regions	Drift I				Drift II			λ	Gal. lat.
	$hV \sin \lambda$	hV	λ	Diff.	$hV \sin \lambda$	hV	Diff.		
XV + XII	1.25	1.63	50	+ 0.28	0.35	0.39	— 0.52	63	— 3
VII	0.75	1.55	151	+ 0.20	0.85	0.95	+ 0.04	64	— 7
XVI	1.35	1.45	111	+ 0.10	0.35	0.70	— 0.21	30	+ 8
III	0.65	1.34	29	— 0.01	0.65	0.70	— 0.21	111	+ 9
IX	1.35	1.41	73	+ 0.06	0.35	0.70	— 0.21	150	— 12
XIII	1.75	1.82	106	+ 0.47	0.35	0.83	— 0.08	155	+ 26
XVIII	0.95	0.99	74	— 0.36	0.45	1.07	+ 0.16	25	— 26
II + VI	0.55	1.30	25	— 0.05	1.35	1.49	+ 0.58	115	— 28
X	1.45	1.53	71	+ 0.18	0.55	0.96	+ 0.05	145	+ 40
XVII	0.95	1.01	109	— 0.34	0.35	0.60	— 0.31	36	— 40
IV	1.25	1.33	70	— 0.03	0.85	0.86	— 0.05	98	+ 48
VIII	1.25	1.30	107	— 0.05	1.15	1.17	+ 0.26	80	— 51
XI	0.65	0.92	135	— 0.43	1.55	1.66	+ 0.75	111	+ 60
XIV	0.95	1.39	43	+ 0.04	0.35	0.38	— 0.53	68	— 55
V	1.15	1.27	115	— 0.08	0.95	0.97	+ 0.06	79	+ 61
I	1.25	1.35	68	0	1.15	1.17	+ 0.26	101	— 62

The columns headed Diff. give the differences between the hV's and the mean values of hV from drift I and drift II.

The regions have been arranged according to the galactic latitude. Thus it is easy to see that for drift I the deviations for the five regions that are nearest to the Milky Way, are all positive except one, while for drift II it is just the reverse. The mean hV for these regions is in drift I above, in drift II below the mean hV of all the regions taken together

This seems to point for drift I to an increase and for drift II to a decrease of hV with galactic latitude. Eddington came to the same conclusion for drift I; for drift II he found no indication of change of hV with galactic latitude.

c. Distribution of the stars over the two drifts.

The columns of table XX headed N give the numbers of stars belonging to each drift.

Adding these numbers, we find for drift I 836 stars and for drift II 584 stars. So the proportion in which the stars are divided over the two drifts proves to be 1.43. No indication has been found of any concentration of either drift in a group of adjacent regions.

d. Derivation of the true vertices and the apex and speed of the solar motion.

It will be remembered that we have taken all the motions of the stars relative to the observer, in other words relative to the Sun. As, however, the Sun itself moves, the drift motions which we have found relative to the Sun, do not give an insight into the true motions of the drifts relative to each other.

For that reason we shall remove the motion of the Sun and determine the drift motions relative to a point which we can consider to be a fixed point. We take for it the centre of gravity of the two drifts This centre is in such a position relative to the two drifts that the motions of the two drifts relative to this centre must always be diametrically opposite to each other. So these motions will be directed to two points of the sky diametrically opposite to each other. These points are called the *true vertices.*

To compute these true vertices, we take the apparent antivertices of the drifts, mentioned in the preceding chapter. The lines of sight to these points are inclined at an angle of 99°. Starting from the Sun S, we draw two arrows SD_1 and SD_2 proportional to the speed V_1 and V_2 of the two drifts and inclined at an angle of 99°. As these arrows represent the drift motions, it is possible to find the motion of the centre of gravity of the stars relative to the Sun. For this we divide the line D_1D_2 at C so that $D_1C : CD_2 = 1 : 1.43$. The centre of gravity is C and its motion SC. The solar motion relative to the centre of gravity is, of course, CS and is directed to the solar apex. The drift motions relative to the same point are CD_1 and CD_2, relative to each D_1D_2 and D_2D_1.

With the help of the positions of the apparent antivertices, the values of hV_1 and hV_2 and the numbers of the stars for the two drifts, it is easy to compute the following values:

true vertices R.A. 86°.5 Decl. + 14°.9
R.A. 266°.5 Decl. — 14°.9

speed of the drifts relative to each other 1.74 units.

Solar apex relative to the centre of gravity of the stars used in the computation

R.A. 267°.5 Decl. + 42°

speed of the solar motion towards that centre 0.82 units

CHAPTER VII.

DETERMINATION OF SOLAR VELOCITY FROM RADIAL VELOCITIES OF THE *K*-STARS.

The speeds of the drifts which we have found are still expressed in the theoretical unit $\frac{1}{h}$. In the usual manner we can derive from these the speeds in K.M. For this purpose, we want, however, a value of the solar speed expressed in K.M. We might take for it the value of 19.5 K.M., as has been found from the greatest amount of data But in this case we get a solar speed computed relative to the centre of gravity of those stars whose R.V. has been used.

The solar speed assumed by us has been computed, however, relative to the centre of gravity of the *K*-stars. So we shall also determine the speed in K.M. relative to the same centre of gravity. Therefore we have added to our investigation a computation of the solar speed by means of 528 *K*-stars whose radial velocities are known. For this computation the radial velocities of the *K*-stars have been taken from

a. Lick. Obs. Bull. Vol. VII Numb. 229 pag. 119.
b. Radial Velocities of 500 stars by W. S. Adams.
Astron. Journ. 42 Numb. 2 (Sept. 1915) pag. 172.

We have thought it necessary to correct these radial velocities with an amount of — 2.0 K.M. Campbell gives two values for the correction — 1.91 K.M. and — 2.82 K.M. As the amount of the correction is of little influence on the results, we adopted the correction — 2.0 K M.

For all these 528 *K*-stars we have computed $\varrho - 2$ K.M. and moreover the distance λ from the star to the apex determined by us: R.A. 267°.5 Decl. + 42°.

Thus we found for each star an equation of condition:

$$V \cos \lambda = \varrho - 2.0.$$

These equations treated with least squares gave a solar speed:

$$V = -22.63 \text{ K M.}$$

If we omit 8 stars whose radial velocity amounts to more than 80 K.M., we find:

$$V = -20{,}8 \text{ K M. prob. err. } 1{,}0 \text{ K.M.}$$

The latter value is almost the same as Campbell found from 382 *K*-stars, viz $V = -21.2$ K.M.

Another value was found by him, however, from 366 *K*-stars, viz. $V = -18.9$ K M. For both determinations Campbell assumed an apex R.A. 270° Decl. + 30.

If we take $V = -20{,}8$ K.M., then the solar speed will be $0{,}82 \times \frac{1}{h} = -20{,}8$ K M. So the theoretical unit $\frac{1}{h} = -\frac{20.8}{0{,}82}$ K.M.

This gives for the speeds of the drifts:

Drift I: $V_1 = 1.35 \frac{1}{h} = 34.2$ K.M. prob. err. 1.6 K.M.

„ II: $V_2 = 0.90 \frac{1}{h} = 22.8$ K.M. „ „ 1.1 K.M.

relative speed of the

two drifts $= 1.74 \frac{1}{h} = 44.1$ K.M. „ „ 2.2 K.M.

CHAPTER VIII.

FINAL REMARKS.

Finally a few words about the results of our investigation compared with those which others obtained.

The definite results which we have obtained are the following.

The number of stars used in the investigation is 1420.

Apparent motion	Antivertices.		hV	Velocity K.M. per sec.	No. of stars.
Drift I	R.A. 87°.1 ± 2°.5	Decl. — 15°.9 ± 2°.3	1.35	34.2 ± 1.6	836
II	R.A. 265°.5 ± 3°.4	Decl. — 65°.0 ± 3°.3	0.90	22.8 ± 1.1	585
Relative motion					
true vertex	R.A. 86°.5 266°.5	Decl. + 14°.9 — 14°.9	1.74	44.1 ± 2.2	

$$\frac{\text{Number of stars of drift I}}{\text{Number of stars of drift II}} = 1.43.$$

Solar motion

Apex R.A. = 267°.5 Decl. + 42°.0.

Velocity $= 0.82 \times \frac{1}{h} = -20.8$ K M ± 1.0 derived from rad. vel.

Mean Peculiar Velocity of the stars in the two drifts $\Omega = \frac{1}{h}\sqrt{\frac{4}{\pi}} = 28.98$ K.M.

Let us first of all compare these results with those which Eddington obtained with stars of the same Catalogue, following the same method.

He used 5322 stars, so almost all the material which the catalogue contains.

Eddington found: (all spectra except *B*.)

Apparent motion	Antivertices		hV	Velocity K.M. per sec.
Drift I	R.A. = 90°.8	Decl. = — 14°.6	1.516	31.84
II	R.A. = 287°.8	Decl. = — 64°.1	0.855	17.96
Relative motion				
true vertex	R.A. = 94°.2 R.A. = 274°.2	Decl. = + 11°.9 Decl. = — 11°.9	1.87	39.27

$$\frac{\text{number of stars of drift I}}{\text{number of stars of drift II}} = 1.5$$

Solar motion

$$\text{Apex R.A.} = 267^\circ.3 \quad \text{Decl.} + 37^\circ.4$$

$$\text{Velocity} = 0.91 \times \frac{1}{h} = 19\ 5 \text{ K.M.}$$

If we compare these results with ours, we see that the antivertices differ more in R.A. than in decl.

As regards their position with respect to the galactic plane:

for drift I the antivertex of the *K*-stars is farther from it than the antivertex of Eddington.

for drift II the antivertex of the *K*-stars is nearer to it than the antivertex of Eddington.

The true vertices of the *K*-stars are at 5° from the Milky Way, this is a little farther than the true vertices of Eddington.

The velocities of both drifts of the *K*-stars are greater than with Eddington.

The distribution of the stars over the two drifts does not differ much.

The direction of the Sun's motion deviates 5°.6 in decl. from the direction found by Eddington.

This seems to confirm the opinion that for the older types the apex lies farther north

Boss [1]) came to the same conclusion. From his catalogue he derived:

Type	R.A.	Decl.	No. of Stars
Oe 5 — B 5	274°.4	+ 34°.9	490
B 8 — A 4	270°.0	28°.3	1647
A 5 — F 9	265°.9	28°.7	656
G	259°.3	42°.3	444
K	275°.4	40°.3	1227
M	273°.6	38°.8	222

Dyson and Thackeray obtained the same results from the Groombridge stars:

Type	R.A.	Decl.	No. of Stars
B. A.	269°	+ 23	1100
F. G. K.	273°	+ 37	866

The velocity of the Sun derived from the *K*-stars — 20.8 K M. is greater than the velocity — 19.5 K.M which is commonly assumed for all the stars taken together. This agrees with Campbell's velocities for the different types

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[1]) Astron. Journ. 623—24.

He found, taking for the solar apex R A = 270° and D = + 30°

Type	Velocity	No of stars
B	— 20.2	225
A	— 16.8	177
F	— 15.8	185
G	— 16.0	128
G	— 13.9	123
K	— 21.2	382
K	— 18.9	369
M	— 22.6	73
M	— 20.2	70

The much greater value for the Sun's velocity 24 K.M. which has been derived by Boss from all the stars taken together, deserves little credit.

Boss obtained this value by excluding from the stars of his catalogue all the stars with a proper motion larger than a certain amount.

That it is wrong to do this, if one wants to base one's investigation on the random distribution of the peculiar proper motions of the stars, has already been proved by Kapteyn. In Astron Nachr. no. 3487 Kapteyn shows that proper motions may never be excluded because we cannot separate from the observed proper motions the parallactic motion due to the Sun's way through space. If we exclude large total proper motions, the parallactic motion will become too small. If we only take into consideration large proper motions, the parallactic motion will be too large.

It will be useful, however, to discuss this question at greater length, as at first sight the method followed by Boss makes us think that the deviation from the random distribution does not occur in his researches.

Boss supposes all the stars with their proper motions to be brought to the centre O_1 of the area. Each proper motion is represented by an arrow indicating the direction and the amount of this proper motion.

As Boss only wishes to exclude *peculiar* proper motions larger than 20″, he does not omit those arrows whose tops fall outside a circle with a radius of 20″ and O_1 as centre, but he takes as centre a point O_2, to which he assumes point O_1 to have moved in consequence of the apparent parallactic motion due to the Sun's way through space. So point O_1 is moved on the line towards the antapex of solar motion. Now Boss assumes for the distance O_1O_2 3″3 sin. Δ, where Δ is the distance of O_1 from the apex of solar motion.

Boss therefore supposes that as a result of the parallatic motion *all* the stars move from O_1 to O_2 and that consequently the residual motions are the peculiar proper motions. The parallactic motions of stars at a great distance differ however too much from those of stars which are nearer to us, to justify the assumption of a mean parallactic motion.

That this objection is not merely theoretical is evident from the conclusions to which his investigation leads.

For the two groups into which Boss divides the stars of his catalogue he finds:

	No. of Stars	mean magn.	mean prop. mot μ_0	parall. mot. M
Group I	5413	5.7	5″.38	3″.85
II	559	5.3	31″.9	21″.58

These parallactic motions found by Boss show deviations, as Kapteyn had already predicted.

If we compute the mean parallax of stars with mean magnitude and mean prop. mot. which Boss gives for his two groups, according to Kapteyn's Public. of the Astron. Lab. at Groningen no. 8, we shall find

	magn.	μ_0	M	$\bar{\pi}$
Group I	5.7	5″.38	3″.85	0.012
II	5.3	31″.9	21″.58	0.045

$\bar{\pi}$ is the annual parallax computed according to Table G of Publ. no. 8.

From this we easily derive:

for group I centennial parall. mot. 3″.85 = 321 × parall. 0.012
II 21″.58 = 479 × parall. 0.045.

If we compare this with the usual assumption: parallactic motion = 410 × parallax, we see that group I actually gives a solar motion through space that is too small and group II one that is too large.

This clearly shows the truth of Kapteyn's assertion that the exclusion of proper motions is never permitted.

STELLINGEN.

I.

Een nauwkeurig onderzoek naar een derden stroom in de richting van de grootste verschillen tusschen observatie en theorie, zooals men die volgens de methode van Eddington in ieder gebied verkrijgt, is gewenscht.

II.

De door Boss gebruikte afleiding voor de waarde van de zonsnelheid verdient geen vertrouwen.

Astronom. Journal, Vol. 26, no. 614.

III.

De bevestiging van het Julius-effect bij anomale dispersie van het zonnespectrum, gegeven door Albrecht, is niet overtuigend.

Astrophys. Journal, Vol. 41 no. 5, Juni 1915.

IV.

Bij een onderzoek van de systematische bewegingen der sterren volgens de methode van Eddington kan men ook met een aantal sterren, aanmerkelijk kleiner dan Eddington wenschelijk acht, nog wel betrouwbare resultaten verkrijgen.

Monthly Notices, Vol. 17, Nov. 1906.

V.

De door Rasch opgemerkte overeenkomst tusschen de differentiaalvergelijking voor de afhankelijkheid der lichtintensiteit van de temperatuur van een stralend lichaam en de vergelijking van van 't Hoff kan niet verklaard worden op de door hem aangegeven physiologische gronden.

Annalen der Physik IV, Band 14 1904, bladz. 194.

VI.

De door Lux gemaakte opmerking, dat bij toenemende temperatuur de photometrisch gemetene lichtsterkte van een zwart lichaam eindelijk niet meer toeneemt, omdat voor ons oog de „obere Schwellenwert für die Lichtempfindung" bereikt wordt, is onjuist.

Zeitschrift für Beleuchtungswesen, Jan. 1914, bldz. 16.

VII.

Het bewijs van Lobatto voor de stelling, dat iedere hoogere machtsvergelijking minstens één wortel heeft, is onnauwkeurig, waar hij aanneemt, dat, als bij de afbeelding van de punten van het X Y vlak op het U.V.vlak, dit laatste niet geheel bedekt wordt, er noodzakelijk een grens in het U.V.vlak moet zijn, aan welker eene zijde geen correspondeerende punten zijn.

Lobatto's Lessen der Hoogere Algebra 1904, bldz. 120.

VIII.

De oorzaak der helderheidsafname van het midden naar den rand der zonneschijf is onbekend.

IX.

De definitie van hoek, zooals ze in de meeste leerboeken der planimetrie gevonden wordt, dient vervangen te worden door een definitie, waarbij op het verschil in richting van de beide beenen gelet wordt.

X.

„The reciprocal of a binode is called a bitrope; but it is a bitrope of a special kind".

Deze laatste opmerking is niet in overeenstemming met de definitie van bitroop.

Basset. A treatise on the geometry of surfaces 1910, bldz. 24.

XI.

In het „Weekblad voor Gymnasiaal en Middelbaar Onderwijs" van 2 Dec. 1915, blz. 469, schrijft F. Ehrenfest—Afanassjewa: „Derhalve schijnt het wel het doelmatigste de systematiese meetkundecursus aan de middelbare school zo op te bouwen, dat men zich in 't geheel niet bekommert om de vermindering

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van het aantal axioma's en iedere stelling, die de leerlingen als vanzelfsprekend beschouwen, in het eerst zonder meer opneemt als grondslag — dus in deze als axioma — voor de verdere gevolgtrekkingen".

Deze conclusie is uit een didaktisch oogpunt niet te aanvaarden.

XII.

Het door Derksen en de Laive gegeven bewijs, dat bij twee symmetrische viervlakken, de hoogtelijnen op gelijkstandige zijvlakken gelijk zijn, omdat de hoogtelijnen van het eene viervlak gelijk zijn aan de gelijkstandige hoogtelijnen van het symmetrische van het tweede, is niet juist.

Derksen en de Laive. Leerboek der Stereometrie, 4e druk, bldz. 86.

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